ECE 171A: Linear Control System Theory
Lecture 20: PID control (II)

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Reading materials: Ch 11.3, 11.4

Outline

PID Tuning

Integral Windup

Midterm II

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PID Tuning

Users of control systems are frequently faced with the task of adjusting the controller parameters to obtain a desired behavior.

- **Approach 1**: go through the conventional steps of modeling and control design (Lecture 19).
- **Approach 2**: Since the PID controller has so few parameters, special *empirical methods* have been developed for direct adjustment of the controller parameters.

$$u = k_{\mathrm{p}} e + k_{\mathrm{i}} \int_0^t e(\tau) d\tau + k_{\mathrm{d}} \frac{de}{dt} = k_p \left(e + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de}{dt} \right).$$

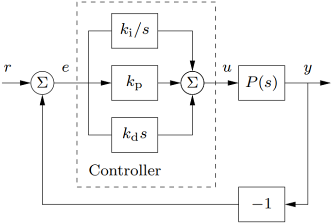


Figure: PID using error feedback

- **Ziegler-Nichols' Tuning**
- **Tuning based on the FOTD model**
- Relay Feedback (Automatic tuning; not required in this course)

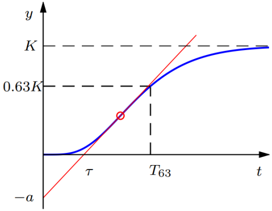
Ziegler-Nichols' Tuning

- Developed by Ziegler and Nichols in the 1940s;
- Very simple to use; Had a huge impact and were adopted by manufacturers of controllers for routine use.

Their idea was to perform a **simple experiment** on the process and **extract features** of process dynamics in the time and frequency domains.

Time-domain method – Bump test

- The process is brought to steady state,
- The input is then changed by a suitable amount,
- The output is measured and scaled to correspond to a unit step input.



(a) Step response method

Two parameters a and τ — the intercepts of the steepest tangent of the step response with the coordinates

Ziegler-Nichols' Tuning

Original Ziegler–Nichols tuning rules

Type	k_p	T_i	T_d
P	$1/a$		
PI	$0.9/a$	$\tau/0.3$	
PID	$1.2/a$	$\tau/0.5$	0.5τ

(a) Step response method

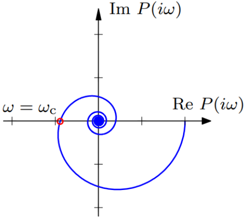
Type	k_p	T_i	T_d
P	$0.5k_\mathrm{c}$		
PI	$0.45k_\mathrm{c}$	$T_\mathrm{c}/1.2$	
PID	$0.6k_\mathrm{c}$	$T_\mathrm{c}/2$	$T_\mathrm{c}/8$

(b) Frequency response method

Frequency-domain method – Critical gain and critical period

- A PID controller is connected to the process; Set $k_\mathrm{i} = 0, k_\mathrm{d} = 0$
- and the proportional gain is increased until the system starts to oscillate.
- **Two parameters** k_c and T_c — The critical proportional gain k_c and the period of oscillation T_c.

$$\omega_c = 2\pi/T_c.$$



(b) Frequency response method

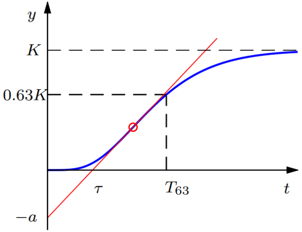
FOTD model

- The Ziegler–Nichols methods use only two parameters to characterize process dynamics.
- Tuning of PID controllers can be improved if we use more parameters.

The first-order and time-delay (FOTD) model

$$P(s) = \frac{K}{1+sT}e^{-\tau s}, \qquad \tau_{\mathrm{n}} = \frac{\tau}{T+\tau}.$$

- $\tau_{\mathrm{n}} \in (0,1)$: relative time delay or normalized time delay.
- **lag dominated** if τ_{n} is close to zero; **delay dominated** if if τ_{n} is close to 1;



(a) Step response method

Bump test

- Zero frequency gain K — the steady-state value
- The time delay τ — the same
- The time T_{63} is the time where the output has reached 63% of K.

$$T = T_{63} - \tau.$$

Example: Atomic force microscope

Example

A simplified model of the dynamics of the vertical motion of an atomic force microscope in tapping mode is

$$P(s) = \frac{1 - e^{-sT_n}}{sT_n(s+1)}.$$

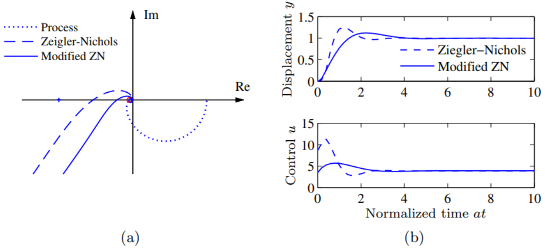


Figure 11.8: PI control of an AFM in tapping mode. Nyquist plots (a) and step responses (b) for PI control of the vertical motion of an atomic force microscope in tapping mode. Results with Ziegler–Nichols tuning are shown by dashed lines, and modified Ziegler–Nichols tuning is shown by solid lines. The Nyquist plot of the process transfer function is shown by dotted lines.

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Integral Windup — Overview

- Many aspects of a control system can be understood from linear models.
- However, some nonlinear phenomena must be taken into account

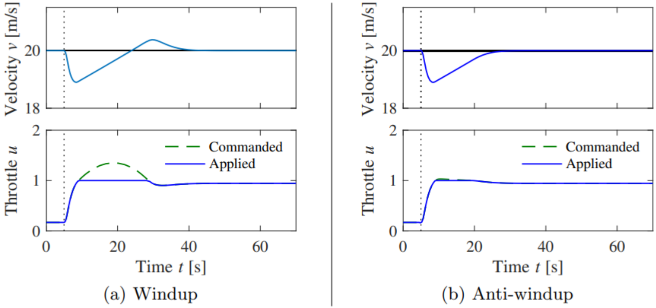
Limitations in the actuators

- e.g., a motor has limited speed, a valve cannot be more than fully opened or fully closed, etc.
- Control variable may reach the actuator limits.
  - when this happens, *the feedback loop is broken*
  - the system runs in *open loop* because the actuator remains at its limit as long as it remains saturated.
- The integral term $\int_0^t e(\tau)d\tau$ will build up since the error $e(t) \neq 0$.
- The control signal will remain saturated even when the error changes
- It may take a long time before the integrator and the controller output come inside the saturation range — **large transients**

This situation above is referred to as **integrator windup**.

Example: Cruise control

When a car encounters a hill that is so steep (6°) that the throttle saturates when the cruise controller attempts to maintain speed.



(a) Windup

(b) Anti-windup

Figure 11.10: Simulation of PI cruise control with windup (a) and anti-windup (b). The figure shows the speed v and the throttle u for a car that encounters a slope that is so steep that the throttle saturates. The controller output is a dashed line. The controller parameters are $k_\mathrm{p} = 0.5$, $k_\mathrm{i} = 0.1$ and $k_\mathrm{aw} = 2.0$. The anti-windup compensator eliminates the overshoot by preventing the error from building up in the integral term of the controller.

Avoiding Windup

- Windup can occur in any controller with integral action.
- There are many methods to avoid windup.

One method for PID control is shown in the figure below:

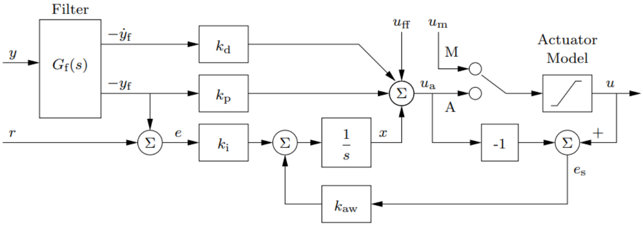
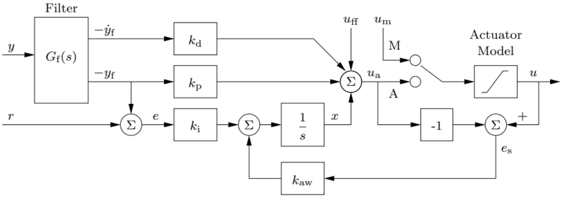


Figure: PID controller with filtering, anti-windup, and manual control. The controller has filtering of the measured signal, an input u_{ff} for feedforward signal, and another input u_m for direct control of the output.

Avoiding Windup



- The system has an extra feedback path from the saturating actuator
- The signal $e_s = u - u_a$
  - $e_s = 0$ if there is no saturation and this extra feedback has no influence
  - When the actuator saturates, $e_s \neq 0$ is fed back to the integrator such that $e_s \to 0$.
- The controller output u will be kept close to the saturation limit.

This anti-windup scheme can also deal with saturation caused by the feedforward signal.

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- HW6 due tomorrow; HW7 will be out on Saturday and due by 11:59 pm, 24 May (Tuesday, Week 9)
- Midterm exam (II) — in class, May 18 (Wednesday in Week 8)
  - **Scope**: Lectures 11 - 21, HW4 - HW6, HW7 (Q1, Q2), DI 5-8; (Reading materials in the textbook)
  - Closed book, closed notes, closed external links.
  - **Come on time** (1 or 2 minutes early if you can; we will start at 9:00 am promptly)
  - No MATLAB is required. No graphing calculators are permitted. A basic arithmetic calculator is allowed.
  - The exams must be done in a blue book. Bring a blue book with you.
  - **No collaboration and discussions are allowed**. It is dishonest to cheat on exams. Instances of academic dishonesty will be referred to the Office of Student Conduct for adjudication. *You don't want to take a risk for such a small thing*.

Midterm II

- Three problems
- Problem 1: True or False
- Problem 2: Bode and Nyquist
- Problem 3: Feedback Control