ECE 171A: Linear Control System Theory
Lecture 23: Loop Shaping

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May 23, 2022

Reading materials: Ch 12.2, 12.3

Outline

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Loop shaping

Loop shaping: choose a compensator $C(s)$ that gives a loop transfer function $L(s) = P(s)C(s)$ with a desired shape. — **Trial and error procedure**

- **Example — Nyquist stability theorem**: To make an unstable system stable we simply have to bend the Nyquist curve away from the critical point $s = -1 + i0$.
- **Method 1** (backward): Determine a loop transfer function that gives a closed loop system with the desired properties and then compute the controller as $C(s) = L(s)/P(s)$. *Drawbacks*:
  - lead to controllers of high order
  - there are limits if the process transfer function $P(s)$ has poles and zeros in the right half-plane,
- **Method 2**: (forward)
  - Start with the process transfer function $P(s)$
  - Change its gain to obtain the desired bandwidth,
  - Add (stable) poles and zeros on $C(s)$ until the desired shape is obtained.

Design considerations

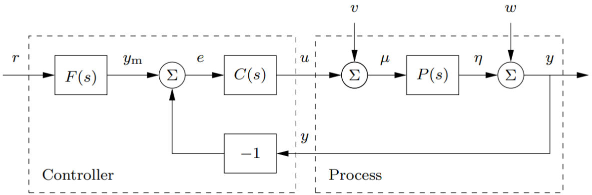


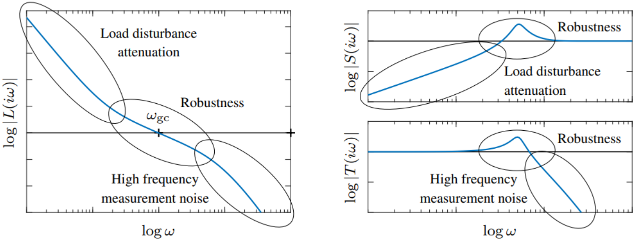
Figure: Block diagram of a control system with two degrees of freedom.

We need a suitable shape for the loop transfer function $L(s) = P(s)C(s)$ that gives good **closed-loop performance** and good **stability margins**.

- Good performance requires that the loop transfer function $L(s)$
  - is large for low frequencies — **good tracking of reference signals**
  - has **good attenuation** of low-frequency load disturbances.
- Since $G_{\mathrm{yw}} = S = 1/(1 + L(s))$ (note that $G_{\mathrm{er}} = S$ if $F(s) = 1$), for frequencies ω where $|L(i\omega)| > 100$
  - disturbances will be attenuated by approximately a factor of 100
  - the steady-state tracking error $|e(t)| = |r(t) - y(t)|$ is less than 1%.

Design considerations

The loop transfer function should thus have roughly the shape shown in the following figure

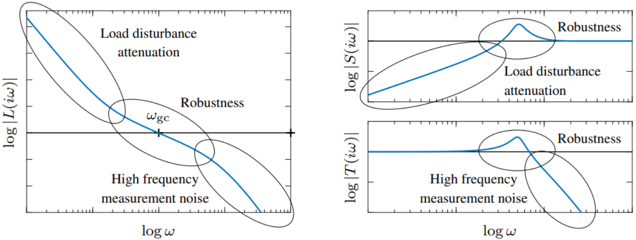


(a) Gain plot of loop transfer function (b) Gain plot of sensitivity functions

- It has unit gain at the gain crossover frequency ($|L(i\omega_{gc})| = 1$),
- large gain for lower frequencies $\omega < \omega_{gc}$, and
- small gain for higher frequencies $\omega > \omega_{gc}$

Robustness is determined by the shape of the loop transfer function around the gain-crossover frequency ω_{gc}.

Design considerations



(a) Gain plot of loop transfer function (b) Gain plot of sensitivity functions

- It would be desirable to transition from high loop gain $|L(i\omega))|$ at low frequencies to low loop gain as quickly as possible,
- **Robustness requirements** restrict how fast the gain can decrease:
  - For a minimum-phase system, the relationship between slope n_{gc} and phase margin φ_m (in degrees) is (no need to remember this equation)

$$n_{gc} \approx -2 + \frac{\varphi_m}{90}.$$

- Time delays and poles and zeros in the right half-plane impose further restrictions (Lecture 25/26)

Outline

Loop shaping via Lead and Lag Compensation

Loop shaping is a **trial-and-error** procedure.

- Many specific procedures are available — they all require experience, but they also give **good insight** into the conflicting specifications.

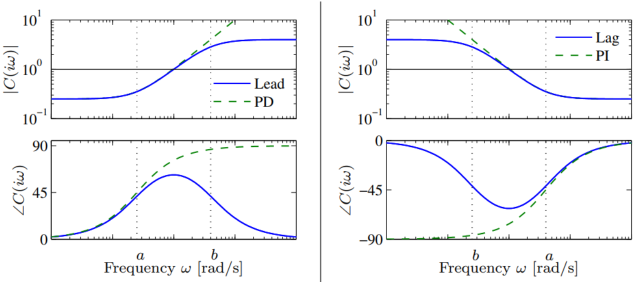
- Start with a Bode plot of the process transfer function $P(s)$
- Choose the gain crossover frequency ω_{gc}
  - A compromise between attenuation of load disturbances and injection of measurement noise.
- Attempt to shape the loop transfer function by changing the **controller gain** and **adding poles and zeros** to the controller transfer function.
  - the **loop gain at low frequencies** can be increased by so-called "**lag compensation**"
  - the **behavior around the crossover frequency** can be changed by so-called "**lead compensation**".
- Different performance specifications are evaluated for each controller.

Lead and Lag Compensation

Simple compensators with transfer function

$$C(s) = k\frac{s+a}{s+b}, \qquad a > 0,\ b > 0$$

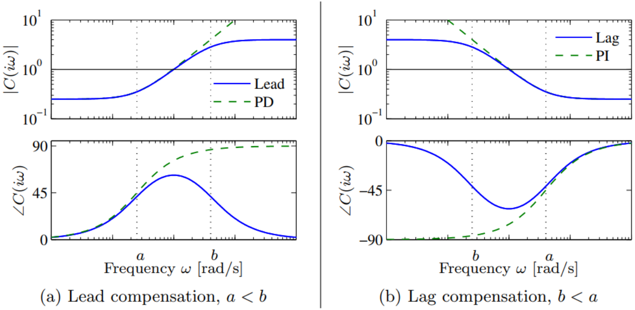
- **Lag compensator** (Phase) if $a > b$; a PI controller is a special case with $b = 0$.
- **Lead compensator** (Phase) if $a < b$; a PD controller with filtering.



(a) Lead compensation, $a < b$

(b) Lag compensation, $b < a$

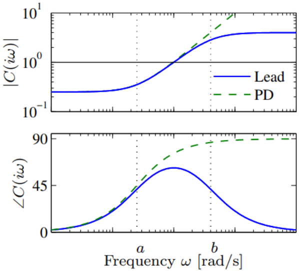
Lead and Lag Compensation



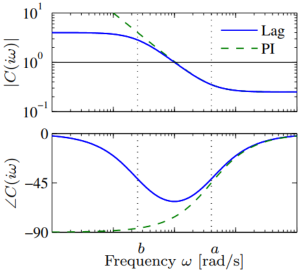
General purpose of **Lag compenstation**

- **increases the gain** at low frequencies
- **improve tracking performance** at low frequencies
- **improve disturbance attenuation** at low frequencies

Lead and Lag Compensation



(a) Lead compensation, $a < b$



(b) Lag compensation, $b < a$

General purpose of **Lead compenstation**

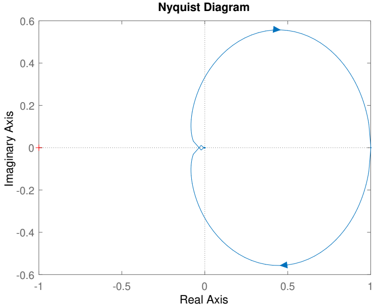
- Add **phase lead** in the frequency range between the pole and zero pair
- By appropriately choosing the location of this phase lead, we can provide **additional phase margin** at the gain crossover frequency.

Example 1

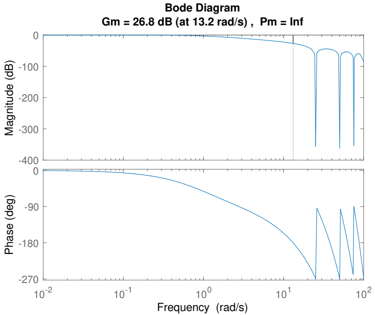
Example (Example 12.4)

The transfer function for the system dynamics is

$$P(s) = \frac{a(1 - e^{-s\tau})}{s\tau(s+a)}, \qquad a = 1, \;\; \tau = 0.25$$



(a) Nyquist plot



(b) Bode plot with margins

Example 1 - unite negative feedback

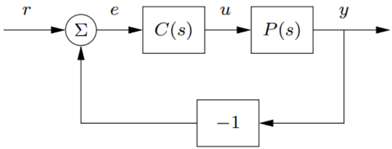
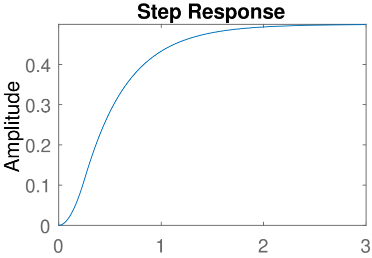
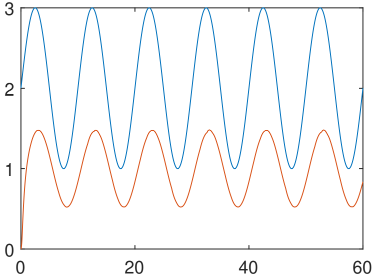


Figure: Unit negative feedback control $C(s) = 1$

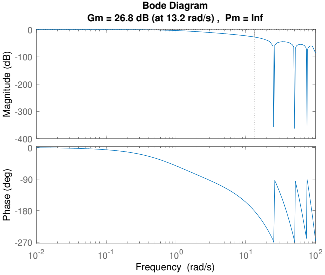


(a) Step response

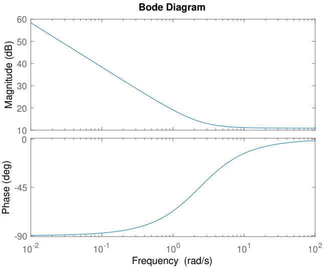


(b) Frequency response with $T = 10$ seconds

Example 1 - Lag compensation



(a) $P(s)$



(b) Lag compensation (PI)

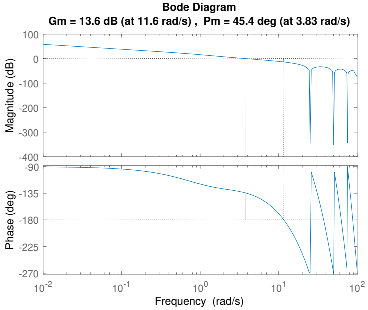


Figure: Margins for $L(s) = P(s)C(s)$

$$C(s) = 3.5 + \frac{8.3}{s}$$

Example 1 - Lag compensation

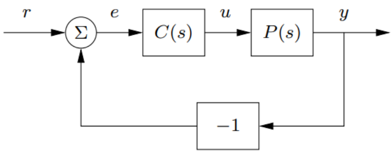
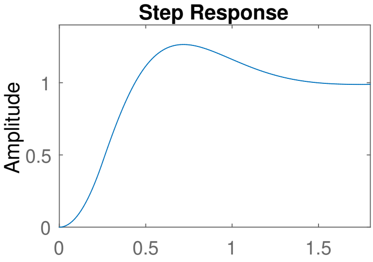
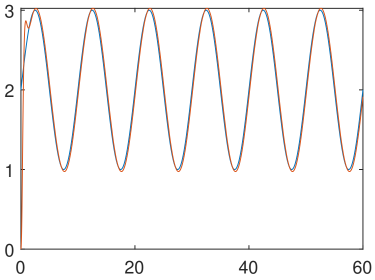


Figure: Feedback control with a lag compensator $C(s) = k_p + \frac{k_i}{s}$



(a) Step response



(b) Frequency response with $T = 10$ seconds

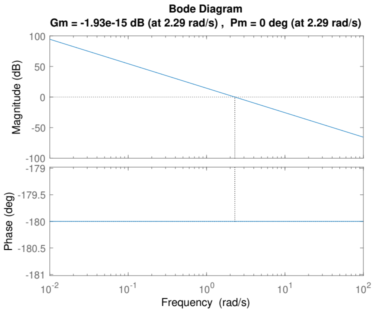
Example 2

Example (Example 12.5)

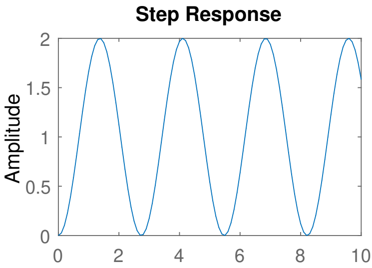
The transfer function for the system dynamics is

$$P(s) = \frac{r}{Js^2}, \qquad r = 0.25, \quad J = 0.0475$$

- less than 1 % error in steady state; $\leq$ 10% tracking error up to 10 rad/s

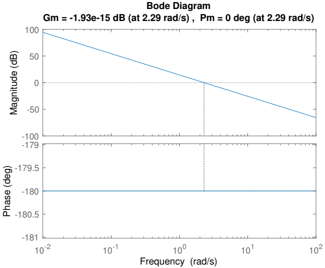


(a) Bode plot with margins

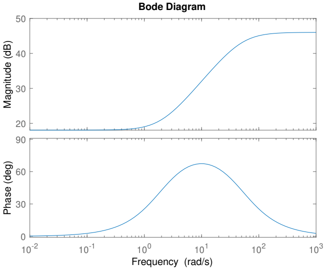


(b) Step response for unit negative feedback

Example 2 - Lead compensation



(a) $P(s)$



(b) Lead compensation

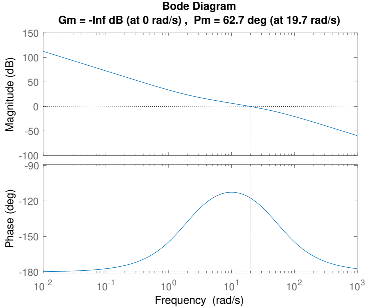
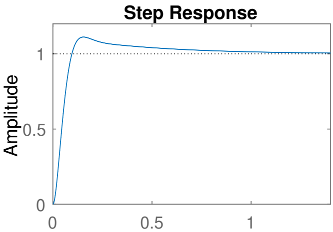


Figure: Margins for $L(s) = P(s)C(s)$

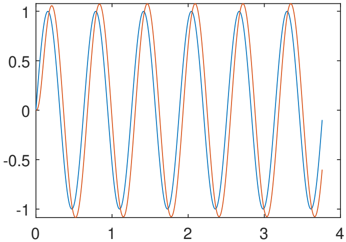
$$C(s) = k\frac{s+a}{s+b},$$

$$a = 2, b = 50, k = 200;$$

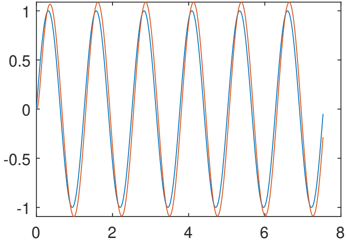
Example 2 - time domain simulations



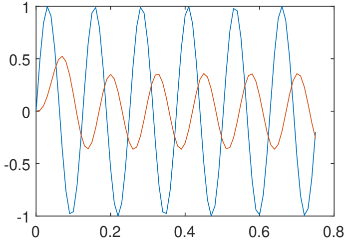
(a) Step response



(b) Frequency response $\omega = 10$



(c) Frequency response $\omega = 1$

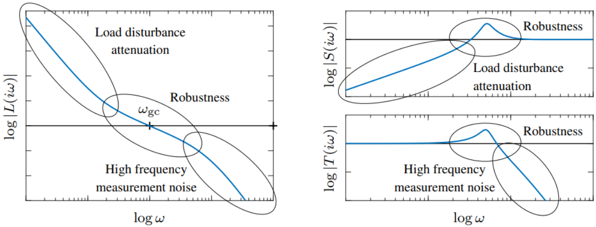


(d) Frequency response $\omega = 50$

Outline

Summary

- The **loop transfer function** should have roughly the shape below



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