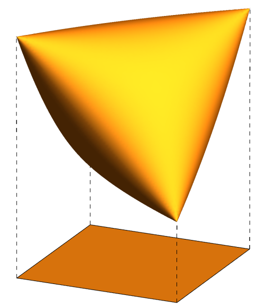
# Semidefinite and Sum-of-squares Optimization (Winter 2023)

**When**: Tuesdays & Thursdays 9:30 am -10:50 am
**Where**: WLH 2111
**Instructor**: Yang Zheng (zhengy@eng.ucsd.edu)
**Course website**: https://zhengy09.github.io/ECE285/ece285.html
**Who**: Graduate students (all disciplines) with an interest in semidefinite and sum-of-squares optimization.



## Course description

Convex optimization has profound impacts on many problems in control theory, discrete and nonlinear optimization, theoretical computer science, and machine learning. It is a fundamental tool to ensure efficient, resilient, and safe operations of many engineering systems, such as smart power grid, transportation, robotics, and many others. Optimization in these areas often takes the form of conic optimization, especially semidefinite programs. This course will cover semidefinite optimization which is a far-reaching generalization of linear programs. Another emphasis of the course will be on sum-of-squares optimization that deals with optimization problems involving polynomials. Some classical applications in control and recent ones in machine learning will be covered too.

A tentative list of topics that we will cover include

- From linear programming to conic programming; Duality theory.
- Semidefinite optimization and convex relaxations.
- Sum-of-squares and moment problems.
- Applications in control, dynamical systems, and machine learning.

**Pre-requisites** This course assumes basic knowledge in linear algebra. Some knowledge of convex optimization will be useful. Mathematical maturity, familiarity with MATLAB, Python, or similar software.

## References

1. Blekherman, G., Parrilo, P. A., & Thomas, R. R. (Eds.). (2012). Semidefinite optimization and convex algebraic geometry. Society for Industrial and Applied Mathematics.
2. Boyd, S., & Vandenberghe, L. (2004). Convex optimization. Cambridge university press.
3. Lasserre, J. B. (2009). Moments, positive polynomials and their applications. World Scientific.
4. Laurent, M. Vallentin, F. (2016) Semidefinite optimization, Lecture notes
5. Ben-Tal, A., & Nemirovski, A. (2001). Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for industrial and applied mathematics.

## Tentative Schedule

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| Week 1 | Jan 10 (L1): Introduction & course logistics | |
| | Jan 12 (L2): Mathematical background | Homework 1 |
| Week 2 | Jan 17 (L3): Review of convexity (I) | |
| | Jan 19 (L4): Review of convexity (II) | |
| Week 3 | Jan 24 (L5): Conic programming | Homework 2 |
| | Jan 26 (L6): LP, QP, QCQP, SOCP, SDP | |
| Week 4 | Jan 31 (L7): Duality in conic programming (I) | |
| | Feb 02 (L8): Duality in conic programming (II) | Homework 3 |
| Week 5 | Feb 07 (L9): Applications of SDPs in control | |
| | Feb 09 (L10): Applications of SDPs in combinatorial problems | Proposal due |
| Week 6 | Feb 14 (L11): Nonconvex quadratic optimization and SDP relaxations | Homework 4 |
| | Feb 16 (L12): Nonnegative polynomials, SOS, and SDP (I) | |
| Week 7 | Feb 21 (L13): Nonnegative polynomials, SOS, and SDP (II) | |
| | Feb 23: Midterm exam | Homework 5 |
| Week 8 | Feb 28 (L14): SOS applications in control & machine learning | |
| | Mar 02 (L15): Dual side: moment problems (I) | |
| Week 9 | Mar 07 (L16): Dual side: moment problems (II) | Homework 6 |
| | Mar 09 (L17): Robust optimization | |
| Week 10 | Mar 14 (L18): Large-scale optimization | |
| | Mar 16 (L19): Review and concluding remarks | |
| Week 11 | Mar 21: Project presentation | |

## Course grade

- 50% homework (6 problem sets; will drop the lowest score)
- 20% midterm exam (in class; one page of cheat sheet allowed)
- 30% final project

These weights are approximate; we reserve the right to change them later.

## Course projects

- **Literature review**: describe in detail a set of 3-5 papers, and reproduce numerical experiments. For example,
  - Different inner/outer approximation of SDPs and their approximation quality.
  - Sparsity-exploiting techniques for SDPs/SOS optimization.
  - First-order algorithms for large-scale SDPs.
  - Basis pursuit or column generation techniques for SDPs.
- **Design methodology**: design a new SDP/SOS methodology for addressing a particular problem (e.g., from your own research). For example,
  - Semidefinite/SOS relaxations for neural network verification.

## Collaboration and Academic Integrity

UCSD's Code of Academic Integrity applies to this course. It is dishonest to cheat on exams, copy other people's work, or fake experimental results. An important element of academic integrity is fully and correctly acknowledging any materials taken from the work of others.

**Homework and project**: The due date of each homework and project assignment will be clearly stated. We expect you to turn in all completed problem sets on time. Late submissions and deadline extensions will not be possible because our schedule is very tight.

**Collaboration policy**: We encourage working together whenever possible: homework sets, lecture notes, general discussions of projects. But please note that the work you turn in should be your own! It is not acceptable to copy a solution that someone else has written. Instances of academic dishonesty will be referred to the Office of Student Conduct for adjudication.


## Acknowledgments

The design of this course is inspired by the following excellent courses

- Topics in Convex Optimisation, University of Cambridge (Lecturer: Hamza Fawzi)
- ORF523: Convex and Conic Optimization, Princeton University (Lecturer: Amir Ali Ahmadi)
- 6.256 Semidefinite Optimization, MIT (Lecturer: Pablo A. Parrilo)