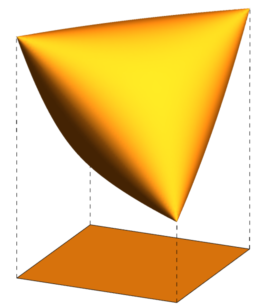
UC San Diego
JACOBS SCHOOL OF ENGINEERING
Electrical and Computer Engineering

Semidefinite and Sum-of-squares Optimization (Winter 2024)



When: Tuesdays & Thursdays 3:30 pm - 4:50 pm
Where: WLH 2207
Instructor: Yang Zheng (zhengy@ucsd.edu)
Course website: https://zhengy09.github.io/ECE285/ece285.html
Who: Graduate students (all disciplines) with an interest in semidefinite, sum-of-squares, and polynomial optimization.

Course description

Convex optimization has profound impacts on many problems in control theory, discrete and nonlinear optimization, theoretical computer science, and machine learning. It is a fundamental tool to ensure efficient, resilient, and safe operations of many engineering systems, such as smart power grid, transportation, robotics, and many others. Optimization in these areas often takes the form of conic optimization, especially semidefinite programs. This course will cover semidefinite optimization which is a far-reaching generalization of linear programs. Another emphasis of the course will be on sum-of-squares optimization that deals with optimization problems involving polynomials. Some classical applications in control and recent ones in machine learning will be covered too.

A tentative list of topics that we will cover include

- From linear programming to conic programming; Duality theory.
- Semidefinite optimization and convex relaxations.
- Sum-of-squares and moment problems.
- Applications in control, dynamical systems, and machine learning.

Pre-requisites This course assumes basic knowledge in linear algebra. Some knowledge of convex optimization will be useful. Mathematical maturity, familiarity with MATLAB, Python, or similar software.

References

1. Blekherman, G., Parrilo, P. A., & Thomas, R. R. (Eds.). (2012). Semidefinite optimization and convex algebraic geometry. Society for Industrial and Applied Mathematics.

2. Boyd, S., & Vandenberghe, L. (2004). Convex optimization. Cambridge University Press.

3. Lasserre, J. B. (2009). Moments, positive polynomials and their applications. World Scientific.

4. Laurent, M. & Vallentin, F. (2016) Semidefinite optimization, Lecture notes

5. Ben-Tal, A., & Nemirovski, A. (2001). Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for industrial and applied mathematics.

Tentative Schedule

Week 1	Jan 09 (L1): Introduction & course logistics	
	Jan 11 (L2): Mathematical background	Homework 1
Week 2	Jan 16 (L3): Review of convexity (I)	
	Jan 18 (L4): Review of convexity (II)	
Week 3	Jan 23 (L5): Conic programming	Homework 2
	Jan 25 (L6): LP, QP, QCQP, SOCP, SDP	
Week 4	Jan 30 (L7): Duality in conic programming (I)	
	Feb 01 (L8): Duality in conic programming (II)	Homework 3
Week 5	Feb 06 (L9): Applications of SDPs in control	
	Feb 08 (L10): Applications of SDPs in combinatorial problems	Proposal due
Week 6	Feb 13 (L11): Nonconvex quadratic optimization and SDP relaxations	Homework 4
	Feb 15 (L12): Iterative inner/outer approximations	
Week 7	Feb 20 (L13): Nonnegative polynomials, SOS, and SDP (I)	
	Feb 22 (L14): Nonnegative polynomials, SOS, and SDP (II)	Homework 5
Week 8	Feb 27: Midterm exam	
	Feb 29 (L15): SOS applications in control & machine learning	
Week 9	Mar 05 (L16): Dual side: moment problems	Homework 6
	Mar 07 (L17): Robust optimization	
Week 10	Mar 12 (L18): Large-scale optimization	
	Mar 14 (L19): Review and concluding remarks	
Week 11	Mar 19: Project presentation	

Course grade

- 50% homework (6 problem sets; will drop the lowest score)
- 20% midterm exam (in class; one page of cheat sheet allowed)
- 30% final project
- Course attendance is mandatory; 5% extra credit for course attendance/participation.

These weights are approximate; we reserve the right to change them later.

Course projects

Projects can have the following different formats (see the course website for more details):

- **Literature review**: describe in detail a set of 3-5 papers, and reproduce numerical experiments. For example,
  - Different inner/outer approximation of SDPs and their approximation quality.
  - Sparsity-exploiting techniques for SDPs/SOS optimization.
  - First-order algorithms for large-scale SDPs.

  - Basis pursuit or column generation techniques for SDPs.

- **Design methodology**: design a new SDP/SOS methodology for addressing a particular problem (e.g., from your own research). For example,
  - Semidefinite/SOS relaxations for machine learning (such as neural network verification).
  - Semidefinite relaxations in Performance Estimation Problems for Nonsmooth and Nonconvex Optimization

Collaboration and Academic Integrity

UCSD's Code of Academic Integrity applies to this course. It is dishonest to cheat on exams, copy other people's work, or fake experimental results. An important element of academic integrity is fully and correctly acknowledging any materials taken from the work of others.

Homework and project: The due date of each homework and project assignment will be clearly stated. We expect you to turn in all completed problem sets on time. Late submissions and deadline extensions will not be possible because our schedule is very tight.

Collaboration policy: We encourage working together whenever possible: homework sets, lecture notes, general discussions of projects. But please note that the work you turn in should be your own! It is not acceptable to copy a solution that someone else has written. Instances of academic dishonesty will be referred to the Office of Student Conduct for adjudication.

Acknowledgments

The design of this course is inspired by the following excellent courses

- Topics in Convex Optimisation, University of Cambridge (Lecturer: Hamza Fawzi)
- ORF523: Convex and Conic Optimization, Princeton University (Lecturer: Amir Ali Ahmadi)
- 6.256 Semidefinite Optimization, MIT (Lecturer: Pablo A. Parrilo)