

Semidefinite and Sum-of-squares Optimization (Winter 2025)

When: Tuesdays & Thursdays 3:30 pm - 4:50 pm

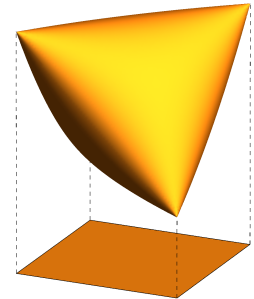
Where: WLH 2207

Instructor: Yang Zheng (zhengy@ucsd.edu)

Course website: <https://zhengy09.github.io/ECE285/ece285.html>

Who: Graduate students (all disciplines) with an interest in semidefinite, sum-of-squares, and polynomial optimization.

(scan me)



Course description

Convex optimization has profound impacts on many problems in control theory, discrete and nonlinear optimization, theoretical computer science, and machine learning. It is a fundamental tool to ensure efficient, resilient, and safe operations of many engineering systems, such as smart power grid, transportation, robotics, and many others. Optimization in these areas often takes the form of conic optimization, especially semidefinite programs. This course will cover semidefinite optimization which is a far-reaching generalization of linear programs. Another emphasis of the course will be on sum-of-squares optimization that deals with optimization problems involving polynomials. Some classical applications in control and recent ones in machine learning will be covered too.

A tentative list of topics that we will cover include

- From linear programming to conic programming; Duality theory.
- Semidefinite optimization and convex relaxations.
- Sum-of-squares and moment problems.
- Applications in control, dynamical systems, and machine learning.

Pre-requisites This course assumes basic knowledge in linear algebra. Some knowledge of convex optimization will be useful. Mathematical maturity, familiarity with MATLAB, Python, or similar software.

References

1. Blekherman, G., Parrilo, P. A., & Thomas, R. R. (Eds.). (2012). Semidefinite optimization and convex algebraic geometry. Society for Industrial and Applied Mathematics.
2. Boyd, S., & Vandenberghe, L. (2004). Convex optimization. Cambridge University Press.
3. Lasserre, J. B. (2009). Moments, positive polynomials and their applications. World Scientific.
4. Laurent, M. & Vallentin, F. (2016) Semidefinite optimization, Lecture notes
5. Ben-Tal, A., & Nemirovski, A. (2001). Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for industrial and applied mathematics.

Tentative Schedule

Week 1	Jan 07 (L1): Introduction & course logistics Jan 09 (L2): Mathematical background	Homework 1
Week 2	Jan 14 (L3): Review of convexity (I) Jan 16 (L4): Review of convexity (II)	
Week 3	Jan 21 (L5): Conic programming Jan 23 (L6): LP, QP, QCQP, SOCP, SDP	Homework 2
Week 4	Jan 28 (L7): Duality in conic programming (I) Jan 30 (L8): Duality in conic programming (II)	Homework 3
Week 5	Feb 04 (L9): Applications of SDPs in control Feb 06 (L10): Applications of SDPs in combinatorial problems	Proposal due
Week 6	Feb 11 (L11): Nonconvex quadratic optimization and SDP relaxations Feb 13 (L12): Iterative inner/outer approximations	Homework 4
Week 7	Feb 18 (L13): Nonnegative polynomials, SOS, and SDP (I) Feb 20 (L14): Nonnegative polynomials, SOS, and SDP (II)	Homework 5
Week 8	Feb 25: Midterm exam Feb 27 (L15): SOS applications in control & machine learning	
Week 9	Mar 04 (L16): Dual side: moment problems Mar 06 (L17): Robust optimization	Homework 6
Week 10	Mar 11 (L18): Large-scale optimization Mar 13 (L19): Review and concluding remarks	
Week 11	Mar 18: Project presentation	

Course grade

- 40% homework (6 problem sets; will drop the lowest score)
- 25% midterm exam (in class; one page of cheat sheet allowed)
- 35% final project/Exam
- Course attendance is mandatory

These weights are approximate; we reserve the right to change them later.

Acknowledgments

The design of this course is inspired by the following excellent courses

- Topics in Convex Optimisation, University of Cambridge (Lecturer: Hamza Fawzi)
- ORF523: Convex and Conic Optimization, Princeton University (Lecturer: Amir Ali Ahmadi)
- 6.256 Semidefinite Optimization, MIT (Lecturer: Pablo A. Parrilo)