① Constrained Optimization Q1

$\nabla f(x) + p \nabla g(x) = 0$

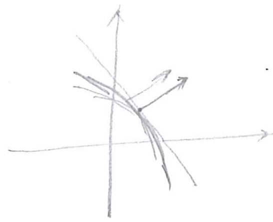
$$\min f(x) \quad \text{s.t. } g(x) = 0 \;\Rightarrow\; L(x,p) = f(x) + p\,g(x)$$

$g(x) = 0 \Rightarrow$ constraint

$$\Rightarrow \begin{cases} \dfrac{\partial L}{\partial x} = 0 \\[2mm] \dfrac{\partial L}{\partial p} = 0 \end{cases}$$

Ⓟ multipliers. geometrical interpretation.

cannot be improve along the feasible domain



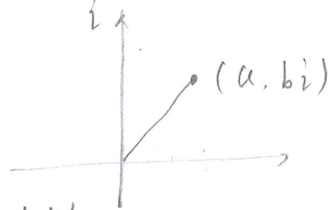
② Parseval's theorem Q2

$$\int_{-\infty}^{+\infty} f^2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |F(w)|^2\,dw$$

$| \; |$: modulus

$f(t)$: time domain

$F(w)$: frequency domain

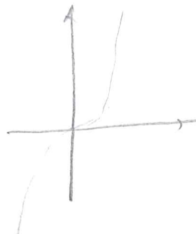


$|a+bi| = \sqrt{a^2+b^2}$

$$\int_{-\infty}^{+\infty} \frac{1}{w^2 + k^2\alpha^2}\,dw = \frac{1}{k\alpha}\int_{-\infty}^{+\infty} \frac{1}{\left(\frac{w}{k\alpha}\right)^2 + 1}\,d\frac{w}{k\alpha} = \frac{1}{k\alpha}\arctan\frac{w}{k\alpha}\Big|_{-\infty}^{+\infty}$$

variable substitution.

$$= \frac{1}{k\alpha}\cdot\pi$$



③ Linear Quadratic Optimal Control.

It is important to derive the correct state space model!

$$\min J = \frac{1}{2}\int_0^{+\infty} x^T Q x + u^T R u\,dt$$

$$\dot{x} = Ax + Bu.$$

$$z = \begin{bmatrix} x \\ p \end{bmatrix} \qquad \dot{z} = Mz$$

1) Derivation $\Rightarrow$ Hamiltonian matrix Q3

$$M = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \qquad P = W_{21}W_{11}^{-1}$$

(stable eigenvalue ; unstable eigenvalue)

$$u = -R^{-1}B^T P x$$

2) Riccati equation $A^T P + PA - PBR^{-1}B^T P = -Q$ Q4 Q5 $P > 0$

$$J_{min} = \frac{1}{2} x^T(0)\,P\,x(0)$$

④ Tracking Problem. Q4 Q6.

⊛ Augmenting the system matrices. (state r)

$$\begin{cases} \dot{x} = Ax + Bu \\ \dot{z} = r - Cx \end{cases} \qquad z = \int (r-y)\,dt$$

define

$\delta x = x_0 - x$

$\delta z = z_0 - z$

$\delta u = u_0 - u.$

equilibrium point

$$0 = Ax_0 + Bu_0$$

$$0 = r - Cx_0$$

$$\frac{d}{dt}\begin{bmatrix} \delta x \\ \delta z \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}\begin{bmatrix} \delta x \\ \delta z \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}\delta u$$

$$\delta y = \begin{bmatrix} C & 0 \end{bmatrix}\begin{bmatrix} \delta x \\ \delta z \end{bmatrix}$$

2) Connections to PID controller Q7