1. Probability ①

Random variable. x. $E(x)$. $Var[x]$ $R_{vv}(\tau) = \sigma_v^2 \delta(\tau)$

Auto-correlation $E[x(t)\, x(t+\tau)]$, white noise , power spectral density is constant

2. Kalman Filter ②. ③ ④

dynamic :

$$\dot{x} = Ax + Bu + F\underline{v} \rightarrow \text{disturbance}$$

$$y = Cx + w \rightarrow \text{noise}$$

observer/ filter

$$\dot{\hat{x}} = A\hat{x} + Bu + \underbrace{k_f}_{\text{a gain / dependence of new measurement}}(y - C\hat{x})$$

$$\hat{y} = C\hat{x} + w$$

$$\min_{k_f} E[(x-\hat{x})(x-\hat{x})^T]$$

W : co-variance of w

V : co-variance of v.

X : 0 unrelated w. v

$$\dot{P} = AP + PA^T - PC^T W^{-1} C P + FVF^T$$

steady-state $\dot{P} = 0$ $k_f = PC^T W^{-1}$

3. Properties ④

1) Frequency response $y \rightarrow \hat{y}$ $G_{kf}(s) = C \cdot (sI - A + k_f C)^{-1} k_f$

2) duality between LQR and kalman Filtering $A^T P + PA - PBR^{-1}B^T P + Q = 0$

4. LQG design ⑤

$$\begin{cases} \dot{x} = Ax + Bu + Fv \\ y = Cx + w \end{cases}$$

$$J = E\left[\int_0^\infty (x^T Q x + u^T R u)\, dt\right]$$

V.S. LQR

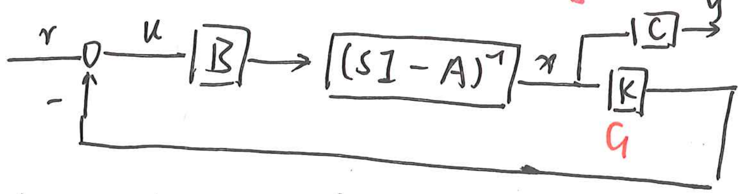
$$\dot{x} = Ax + Bu$$

$$J = \int_0^\infty (x^T Q x + u^T R u)\, dx$$

$u = -\underline{G}x \rightarrow$ LQR gain

$u = -G\hat{x}$

$\hat{x}$ is the estimation of x



① Separation Theorem.

② Cannot guarantee stability margin

$L(s) = K(sI-A)^{-1}B$

K (G)

$L_{lqg}(s) = G(sI - A + BG + k_f C)^{-1} k_f \cdot C(sI-A)^{-1}B$

5. Loop Transfer Recovery ⑥

$F = B$, $V = V_0 + q^2 I$ $W = W_0$

$$\lim_{q \rightarrow \infty} L_{lqg}(s) = L_{lqr}(s)$$