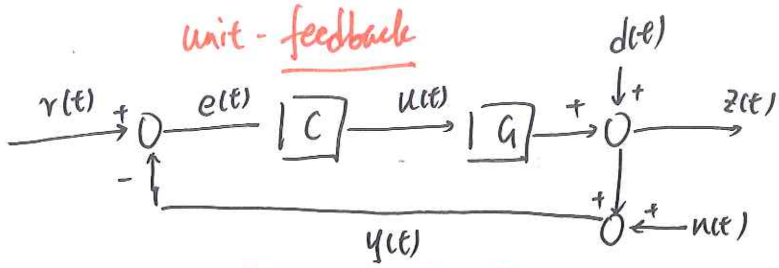
1 Modelling ①

① ODE
② transfer function ✓ } ⇔ linear ODE
③ impulse response
④ state space model } ✓

unit-feedback



Signals in closed-loop system

key point : roots of ODE = poles of ② = eigenvalues of the A matrix

$$G(s) = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \Rightarrow x = \begin{bmatrix} \frac{d^{n-1}y}{dt} \\ \vdots \\ y \end{bmatrix} \quad \frac{dx}{dt} = Ax + Bu.$$

$$y = Cx + Du.$$

state transformation $\hat{x} = T x$ controllable canonical form

2 Analysis : stability / BIBO ②. ③. ④

① all of the poles of the transfer function lie in the open left half plane : $Re(\lambda_i) < 0$

② Nyquist stability criterion : mapping from a complex plane s ⇒ another complex plane.

$$T(s) = \frac{G(s)C(s)}{1 + G(s)C(s)}$$ closed-loop

$L(s) = G(s)C(s)$: open-loop transfer function (why?)

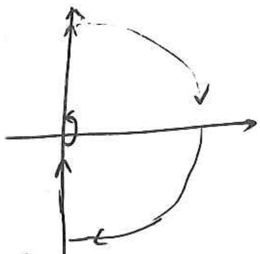
D-contour : enclosing the "whole" of the right half plane

$1 + G(s)C(s) = 0$

⇔ no zeros in the right-half plane.

⇔ the mapping of the D-contour under $L(s)$ encircles $-1 + j0$ ↺ p times

P is the number of poles of L(s) in D-contour



1) positive imaginary axis
2) clockwise semi-circle
3) negative imaginary axis

③ Combining systems.

gain margin / phase margin.

3 Synthesis : { loop shaping (Bode diagram) / robust control (uncertainty) } ⑥. ⑦. ⑧

v.s. pole placement. LQR.

$$z(t) = \frac{GC}{1+GC} \cdot R(s) + \frac{1}{1+GC} D - \frac{GC}{1+GC} N$$

$|T(jw)| = 1$ (low frequency) ↓

$|S(jw)| = 0$ ↓ reject disturbance

↓ 0 (high frequency)

Bode Integral theorem

$$\int_0^\infty \ln |S(jw)| dw = \pi \sum_{k=1}^{M} Re(P_k^u)$$ → unstable poles

(Inherent limitation)

waterbed effect

$|G(jw)C(jw)| > M_L, \quad w < w_L$

$|G(jw)C(jw)| < \frac{1}{M_H}, \quad w > w_H$

Norms and Modelling uncertainty