# Simulations for the second tutorial, B15

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November 22, 2017

Q2

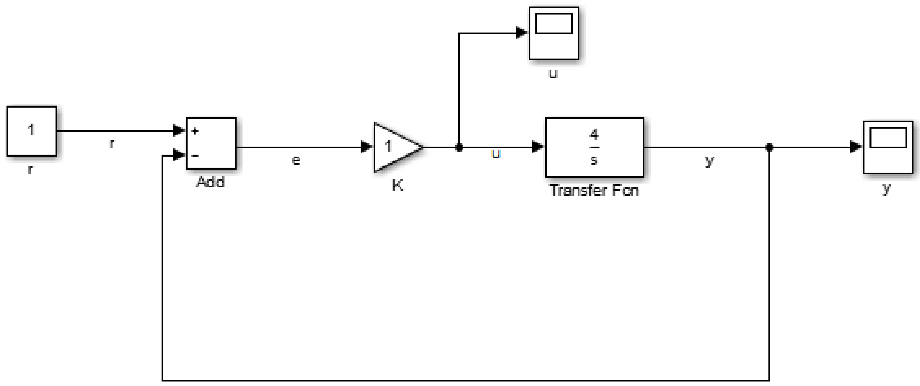



Figure 1: Simulink model for Q2

Case 1: the controller gain $K = 1$

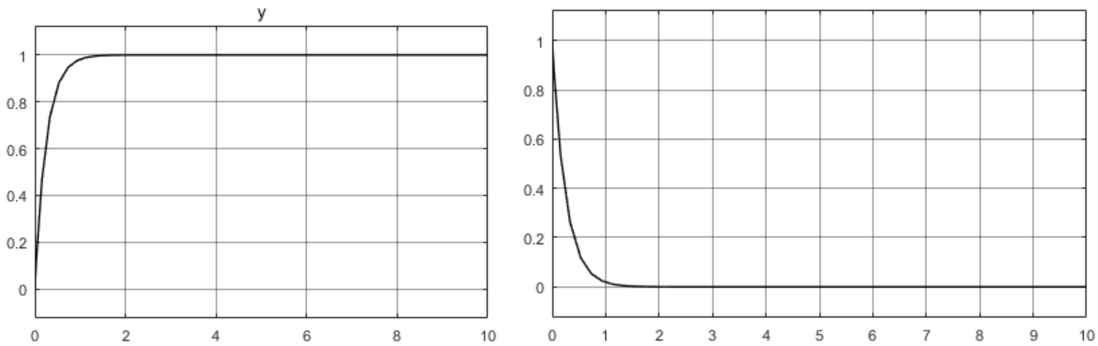



Figure 2: Response when K = 1. Left: output y, Right: control input u

Case 2: controller gain $K = 10$

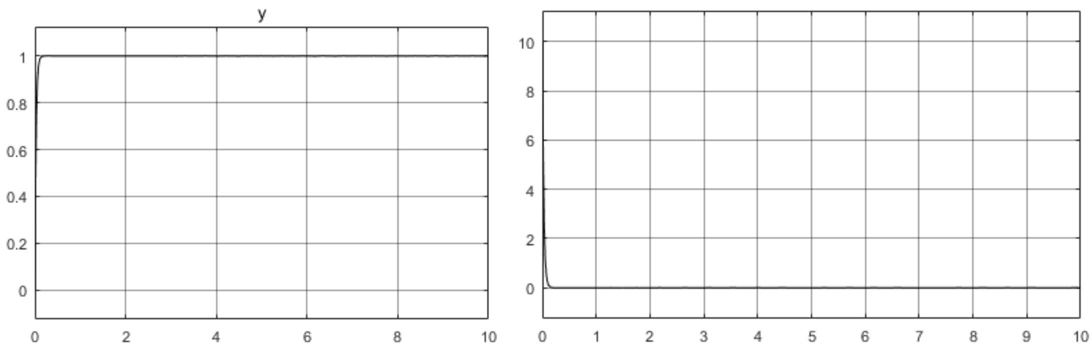
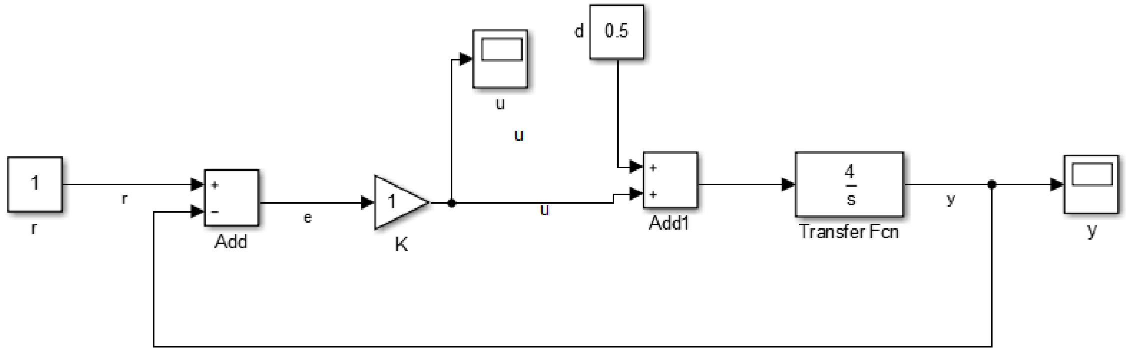



Figure 3: Response when K = 10. Left: output y, Right: control input u

Q4:

We assume there is a constant disturbance $d = 0.5$. If there is no integral feedback, then




Case 1 : K = 1

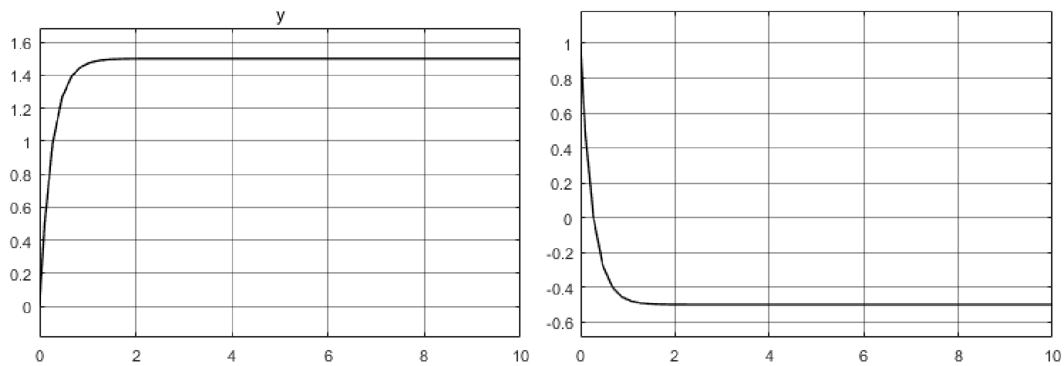



Figure 4: Response when K = 1. Left: output y (non-zero error, around 0.5), Right: control input u (non-zero u, around -0.5)

Case 1 : K = 10

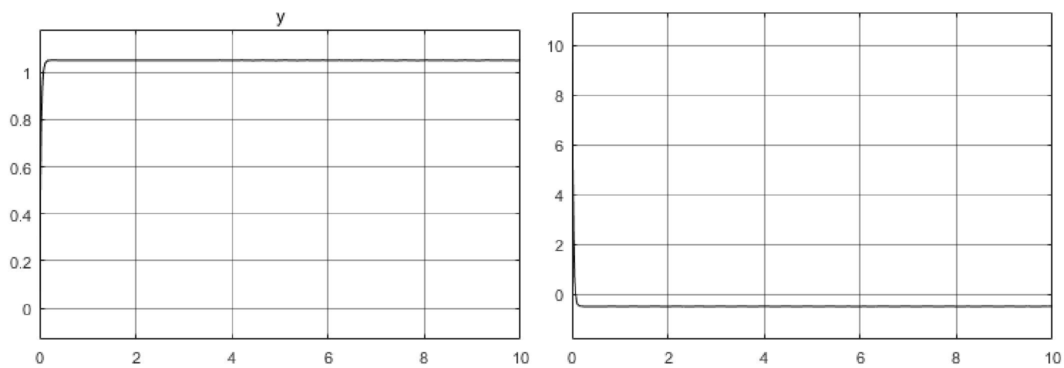
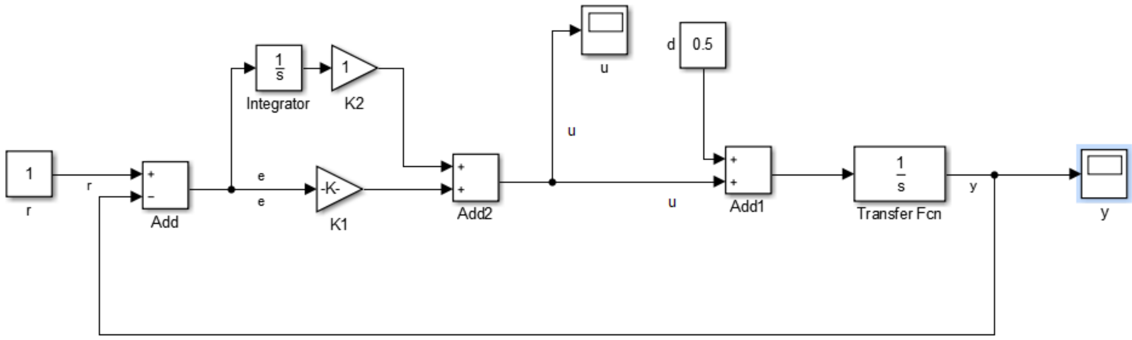



Figure 5: Response when K = 10. Left: output y (non-zero error, around 0.1), Right: control input u (non-zero u, around -0.1)

If there is an integral feedback, then




Feedback gains: $K_1 = \sqrt{3}, K_2 = 1$

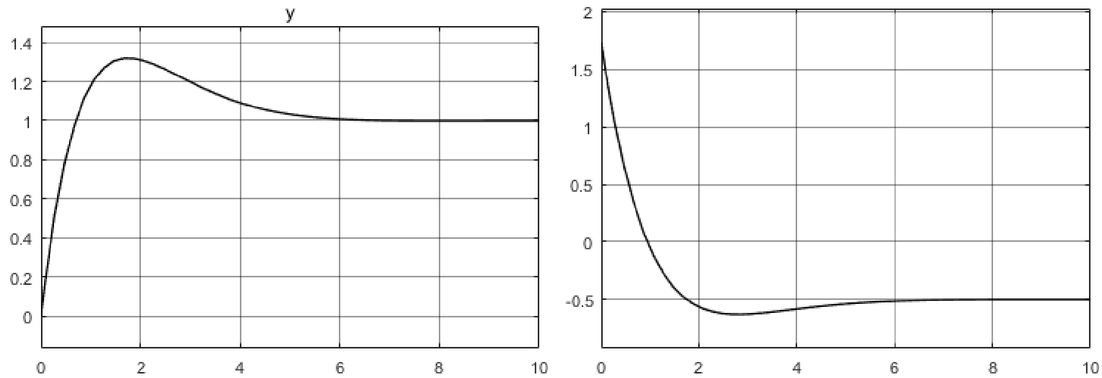



Figure 5: Response when there exists a constant disturbance $d = 0.5$. Left: output y (zero error), Right: control input u (non-zero u, around -0.5, which cancels the disturbance $d$)

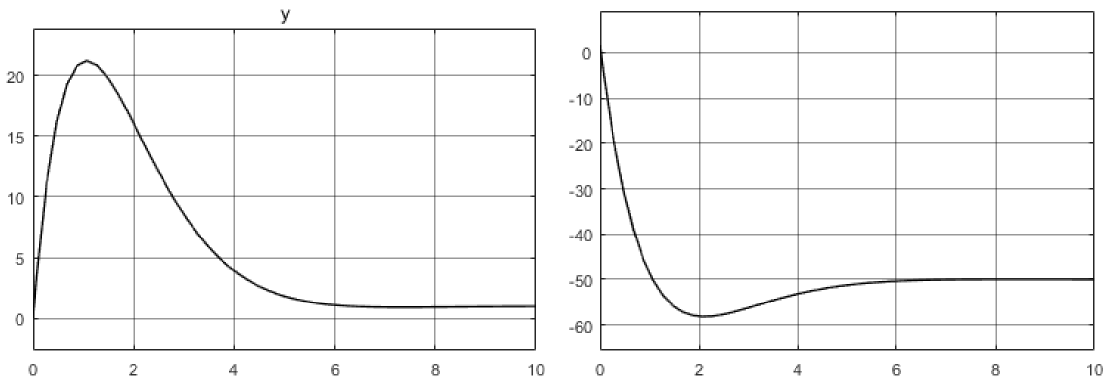



Figure 5: Response when there exists a constant disturbance $d = 50$. Left: output y (zero error), Right: control input u (non-zero u, around -50, which cancels the disturbance $d$)

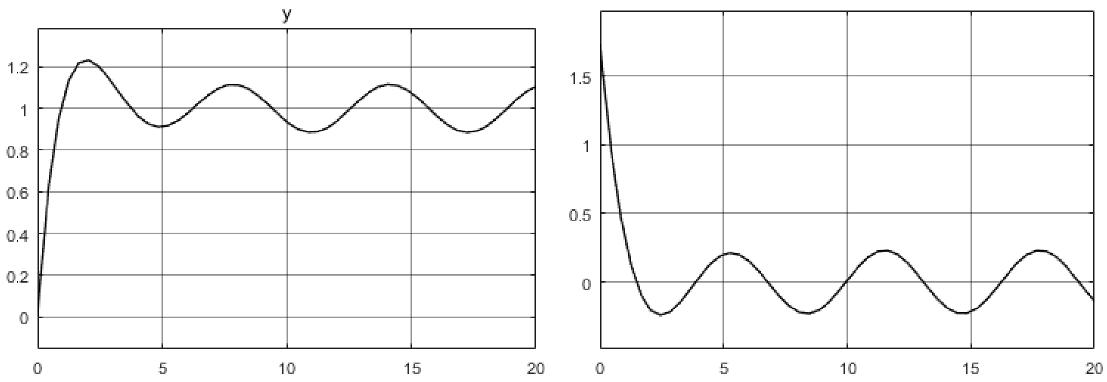



Figure 5: Response when there exists a sine wave disturbance $d$. Left: output y (non-zero error), Right: control input u (non-zero u)

Q5, Nyquist plot

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sy1 =

  0.5858 s + 1
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  s^2 + 1.414 s

Continuous-time transfer function.
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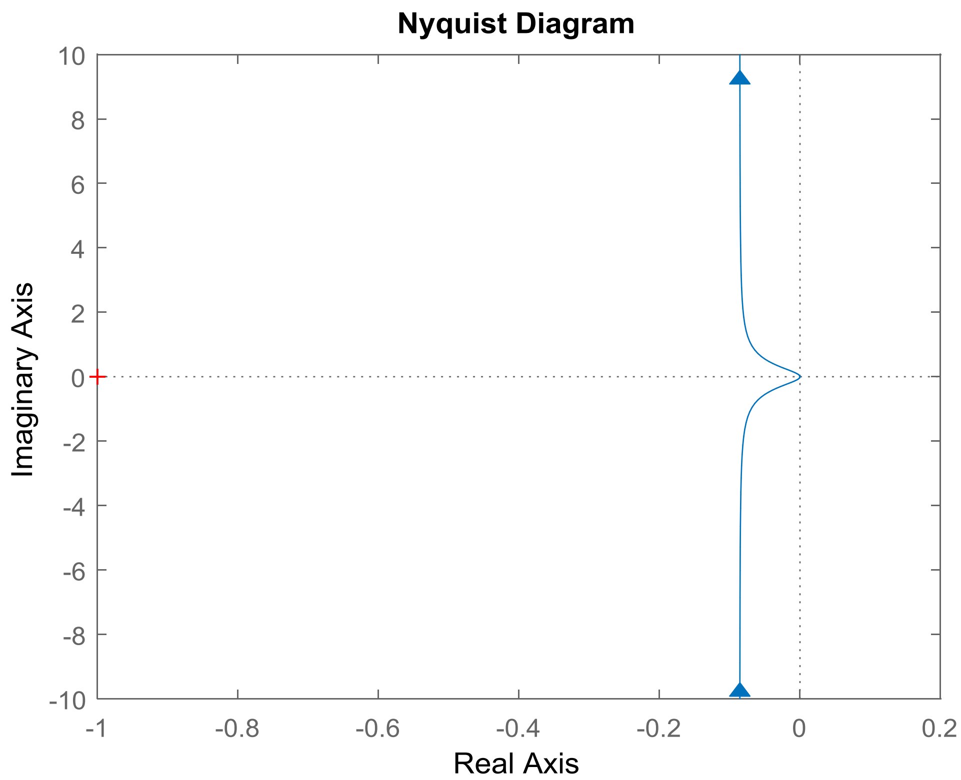
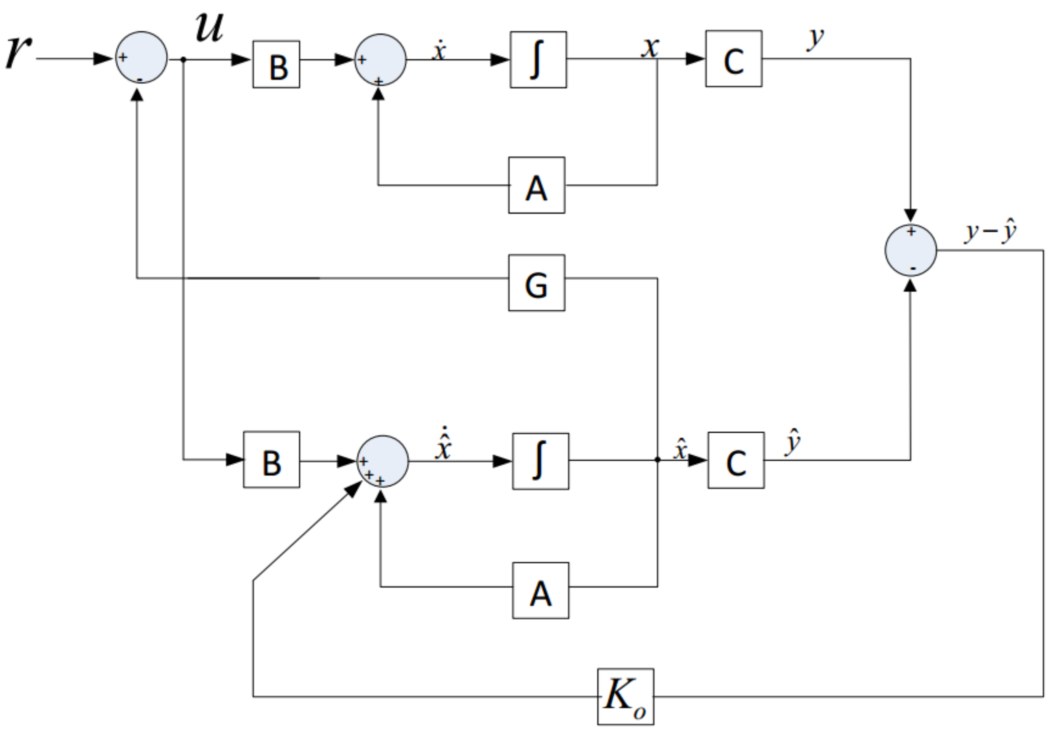



Gm = Inf, Pm = 86.0171

Q6.




Block diagram of closed-loop feedback with observer

Derive a transfer function from $u$ to $v = G\hat{x}$

The observer dynamics:

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + K_0(Cx - C\hat{x})$$

The system dynamics

$$\frac{dx}{dt} = Ax + Bu$$

From the first equation, we have

$$s\hat{X} = A\hat{X} + BU + K_0C(X - C\hat{X}) \Rightarrow (sI - A + K_0C)\hat{X} = BU + K_0CX$$

From the second equation, we have

$$sX = AX + BU \Rightarrow (sI - A)X = BU \Rightarrow X = (sI - A)^{-1}BU$$

Then,

$$(sI - A + K_0C)\hat{X} = BU + K_0C(sI - A)^{-1}BU$$

$$\Rightarrow \hat{X} = (sI - A + K_0C)^{-1}(I + K_0C(sI - A)^{-1})BU$$

$$\Rightarrow \hat{X} = \left((I + K_0C(sI - A)^{-1})(sI - A)\right)^{-1}(I + K_0C(sI - A)^{-1})BU$$

$$\Rightarrow \hat{X} = (sI - A)^{-1}BU$$

This seems to be the same as the transfer function without any observer.

Q7: Connections between LQR and PID control. This is a good example to show that we can use LQR theory to design a PID controller.