

Solution to Q8

Codes:

```
clc;clear
%% SOLUTION TO Q8

% state-space model
A = [-0.0028 0.0389 0 -32.1740;
     -0.065 -0.317 769.2960 0;
     0.0002 -0.001 -0.4272 0;
     0 0 1.0 0];
B = [1.44 0.0008;
     -17.90 -0.0002;
     -1.1579 0;
     0 0];
C = [0 0 0 1];

% controllability test
Qctr = ctrb(A,B)
R_Qctr = rank(Qctr)

% observability test
Qobs = obsv(A,C)
R_Qobs = rank(Qobs)

% Q8-2
B1 = B(:,1);
rank(ctrb(A,B1))

B1 = B(:,2);
rank(ctrb(A,B1))
```

Results:

```
Qctr =
    1.4400    0.0008   -0.7003   -0.0000    2.8225    0.0000    9.7563   -0.0000
   -17.9000   -0.0002  -885.1871    0.0000   675.1778    0.0003   298.1060   -0.0002
    -1.1579     0         0.5128    0.0000    0.6660   -0.0000   -0.9591   -0.0000
         0         0        -1.1579     0         0.5128    0.0000    0.6660   -0.0000

R_Qctr =
     4

Qobs =
         0         0         0     1.0000
         0         0     1.0000         0
    0.0002   -0.0010   -0.4272         0
   -0.0000    0.0008   -0.5868   -0.0064

R_Qobs =
     4
```

```
R_Qctr1 =
    4

R_Qctr2 =
    4
```

Solution to Q9

```
clear; clc;
%% solution to Q9

% state-space model
A = [-0.0028 0.0389 0 -32.1740;
     -0.065 -0.317 769.2960 0;
     0.0002 -0.001 -0.4272 0;
     0 0 1.0 0];
B = [1.44 0.0008;
     -17.90 -0.0002;
     -1.1579 0;
     0 0];

%% design controller
f = [1;0]; % Q 9.b
%f = [0;1] % Q 9.c

% desired poles
p1 = -0.06 + 0.06*1i;
p2 = -0.06 - 0.06*1i;
p3 = -4;
p4 = -5;

% compute Q
Q = [(p1 * eye(4) - A)^(-1)*B*f, (p2 * eye(4) - A)^(-1)*B*f, ...
     (p3 * eye(4) - A)^(-1)*B*f, (p4 * eye(4) - A)^(-1)*B*f]';
k = Q^(-1)*(-ones(4,1))
K = f*k'
```

Results

9. b

Q =

```
1.0e+03 *  
  
0.8393 - 1.2587i -0.9010 - 0.2244i -0.0002 - 0.0001i 0.0028 - 0.0009i  
0.8393 + 1.2587i -0.9010 + 0.2244i -0.0002 + 0.0001i 0.0028 + 0.0009i  
-0.0004 + 0.0000i -0.0594 + 0.0000i 0.0003 + 0.0000i -0.0001 + 0.0000i  
-0.0003 + 0.0000i -0.0365 + 0.0000i 0.0002 + 0.0000i -0.0000 + 0.0000i
```

k =

```
0.0076 + 0.0000i  
-0.0110 + 0.0000i  
-7.0520 + 0.0000i  
-6.7601 + 0.0000i
```

K =

```
0.0076 - 0.0000i -0.0110 - 0.0000i -7.0520 - 0.0000i -6.7601 - 0.0000i  
0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i
```

Qinv =

```
-0.0001 + 0.0005i -0.0001 - 0.0005i 0.0273 + 0.0000i -0.0348 - 0.0000i  
-0.0002 + 0.0001i -0.0002 - 0.0001i -0.0462 + 0.0000i 0.0575 - 0.0000i  
0.0048 - 0.0027i 0.0048 + 0.0027i -11.6401 + 0.0000i 18.6826 - 0.0000i  
0.1497 - 0.0998i 0.1497 + 0.0998i -24.0231 + 0.0000i 30.4837 - 0.0000i
```

9.c

Q =

```
-0.0083 + 0.0025i -0.0014 + 0.0003i -0.0000 + 0.0000i 0.0000 - 0.0000i  
-0.0083 - 0.0025i -0.0014 - 0.0003i -0.0000 - 0.0000i 0.0000 + 0.0000i  
-0.0002 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 0.0000i  
-0.0002 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i -0.0000 + 0.0000i
```

k =

```
1.0e+07 *  
  
0.0002 + 0.0000i  
-0.0034 - 0.0000i  
3.9804 + 0.0000i  
0.1072 + 0.0000i
```

K =

```
1.0e+07 *  
  
0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i 0.0000 + 0.0000i  
0.0002 - 0.0000i -0.0034 + 0.0000i 3.9804 - 0.0000i 0.1072 - 0.0000i
```

Qinv =

1.0e+08 *

-0.0000 + 0.0000i	-0.0000 - 0.0000i	-0.0000 - 0.0000i	0.0000 + 0.0000i
-0.0000 + 0.0000i	-0.0000 - 0.0000i	-0.0008 - 0.0000i	0.0012 + 0.0000i
-0.0006 + 0.0004i	-0.0006 - 0.0004i	1.7682 + 0.0000i	-2.1651 - 0.0000i
-0.0001 + 0.0005i	-0.0001 - 0.0005i	0.0382 + 0.0000i	-0.0486 - 0.0000i