

Matlab Simulations for the Tutorial on Limits of Controller Performance

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1. Solution to Q1

```
% Q1
clc;clear;

% transfer function
sym1_tf = tf([2 4 1],[1 5 2]);

% state space model
sym1_ss = ss(sym1_tf)

% solution by hand
A = [-5 -2;1 0]; B = [1;0]; C = [-6 -3]; D = 2;

% transformation matrix
T = [4 0; 0 4];

T*A*T^(-1)

T*B

C*T^(-1)
```

```
sym1_ss =

  a =
      x1  x2
  x1  -5  -2
  x2   1   0

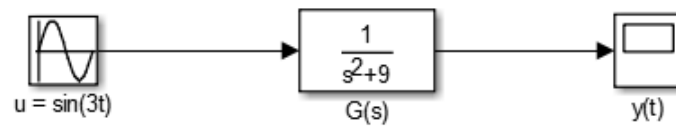
  b =
      u1
  x1   4
  x2   0

  c =
      x1  x2
  y1  -1.5 -0.75

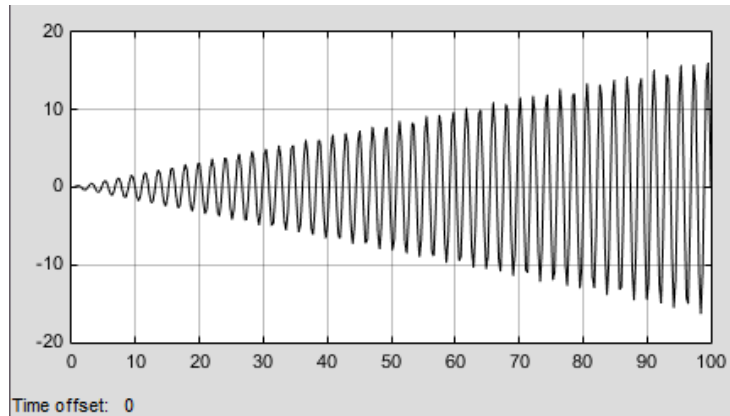
  d =
      u1
  y1   2
```

Continuous-time state-space model.

2. Solution to Q2



Simulink Model



Output response

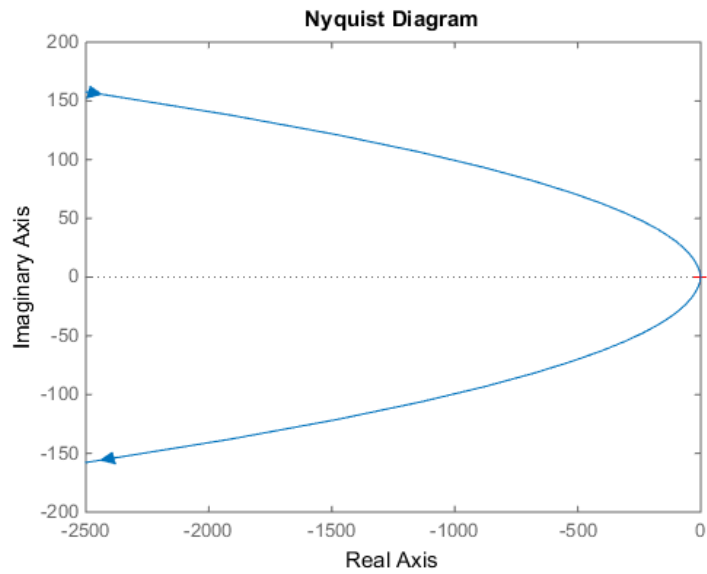
3. Solution to Q3

(Just for verification, you need to know how to draw nyquist plot for simple transfer functions)

```
% Q3
clc;clear;

% transfer function
sym1_tf = tf([10],[1 1 0 0])

% Nyquist Plot
nyquist(sym1_tf)
```



4. Solution to Q5

```

%% Solution to Q5

clc;clear

G_tf = tf([1], [1 -2]); % system transfer function
C_tf = tf([1 -2], [1 3]); % Controller transfer function

% solution to a)
G_ss = ss(G_tf);
C_ss = ss(C_tf);

% solution to b)
CG_ss = series(C_ss,G_ss)

[V,D] = eig(CG_ss.a);

T = inv(V);

CG_ss1 = ss2ss(CG_ss,T)

```

```

CG_ss =

a =
      x1    x2
x1      2  -2.5
x2      0   -3

b =
      u1
x1     1
x2     2

c =
      x1    x2
y1     1     0

d =
      u1
y1     0

```

(a) Original system

```

CG_ss1 =

a =
      x1    x2
x1     2     0
x2     0    -3

b =
      u1
x1     0
x2    2.236

c =
      x1    x2
y1     1  0.4472

d =
      u1
y1     0

```

(b) Transformed system

There is an uncontrollable unstable mode $\lambda = 2$

5. Solution to Q6

```
G_tf = tf([1], [1 5 2 0]);
```

```

%% solution to a)
T_tf = feedback(G_tf,1); %% complementary sensitivity transfer function
[mag1,phase1] = bode(T_tf,1)

zero(1+G_tf) % stable?
pole(T_tf) % stable
margin(G_tf) % Stable?

```

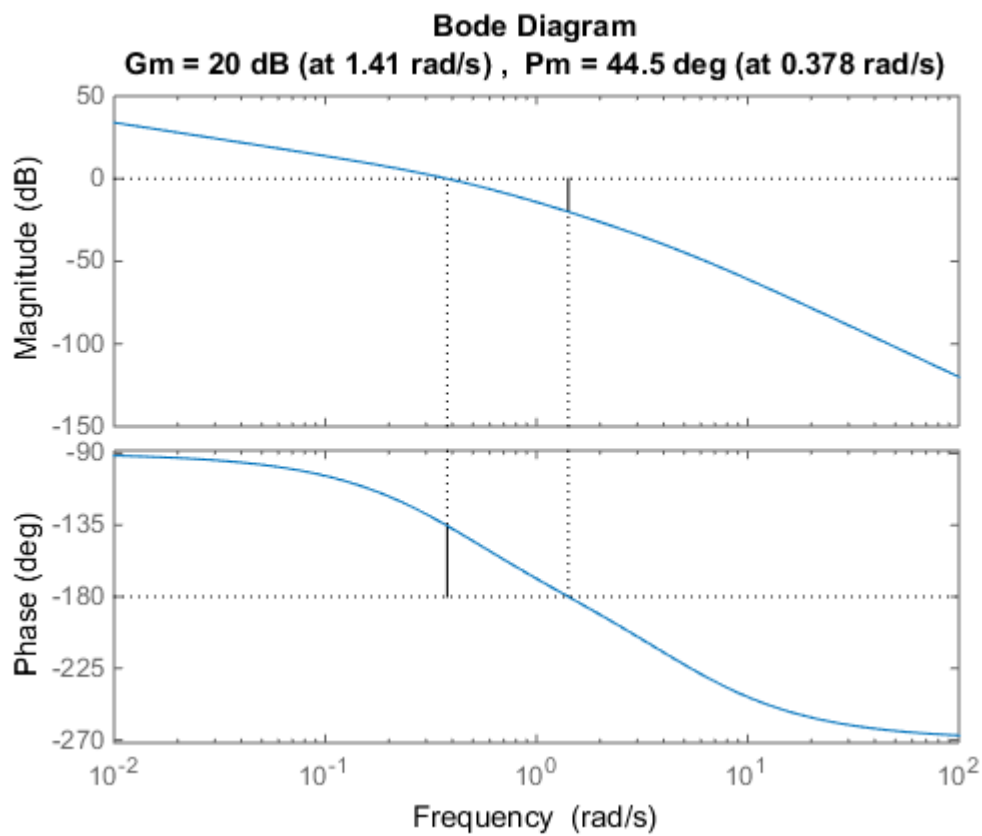
```

mag1 =
    0.2425

phase1 =
   -165.9638

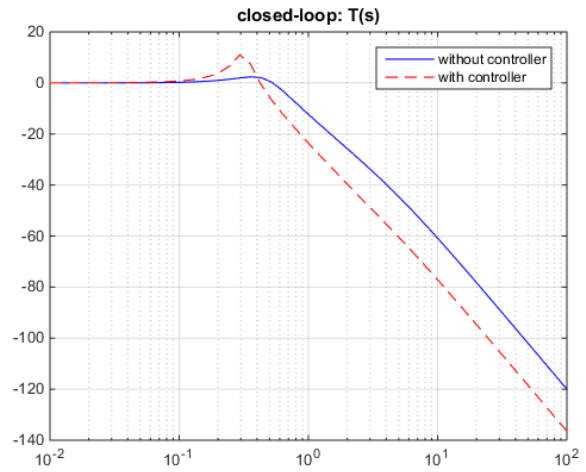
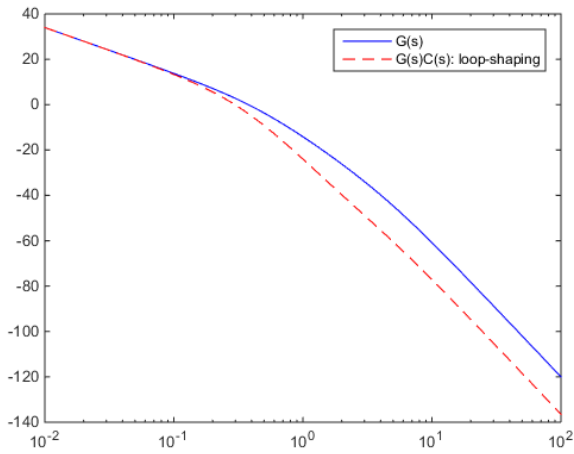
ans =
   -4.6135 + 0.0000i
   -0.1933 + 0.4236i
   -0.1933 - 0.4236i

```

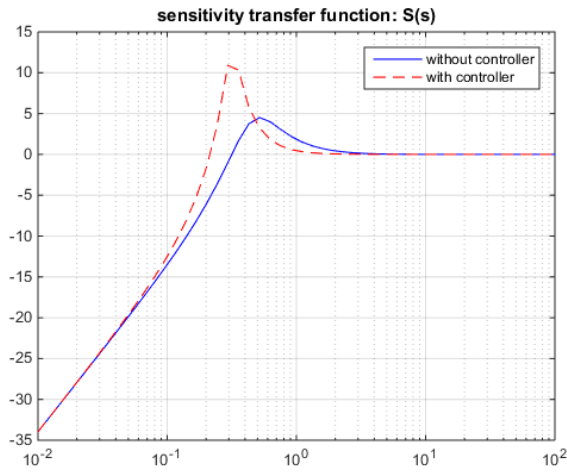


If $K = 0.15$; $a = 2$; $b = 0.3$; then $T(j1) = 0.0664$
 Loop-shaping:

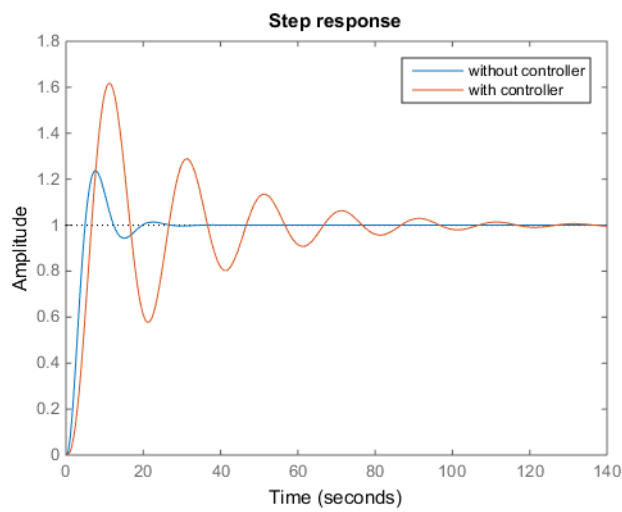
open-loop and closed-loop transfer function



Sensitivity transfer function

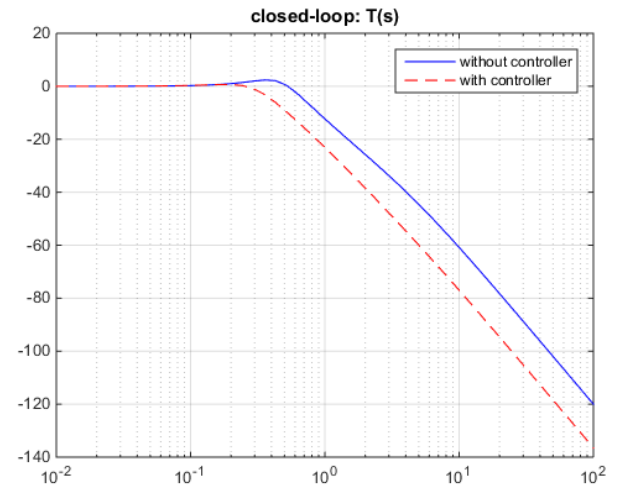
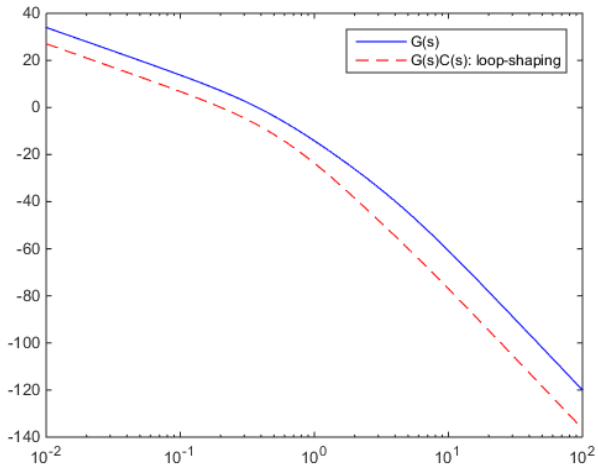


Step-response

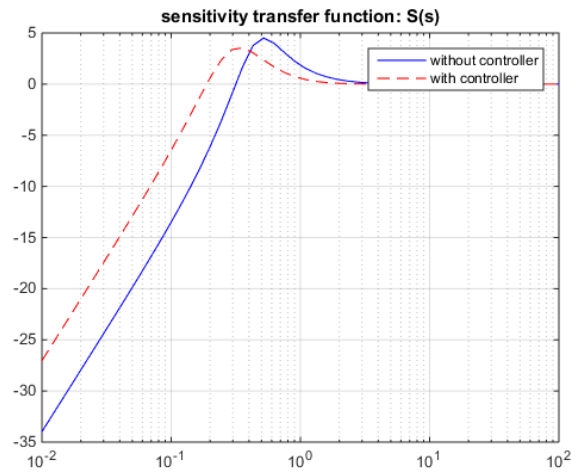


If $\kappa = 0.15$; $a = 3$; $b = 1$; then $T(j1) = 0.0702$
 Loop-shaping:

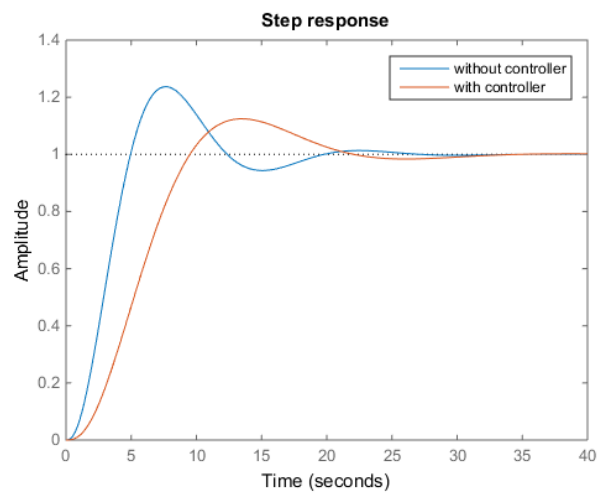
open-loop and closed-loop transfer function

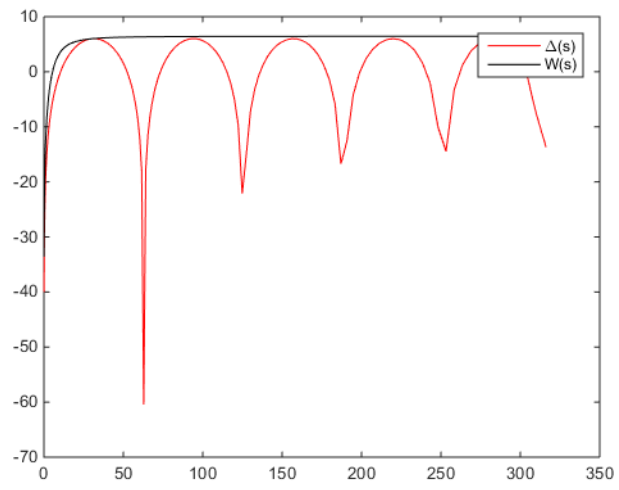
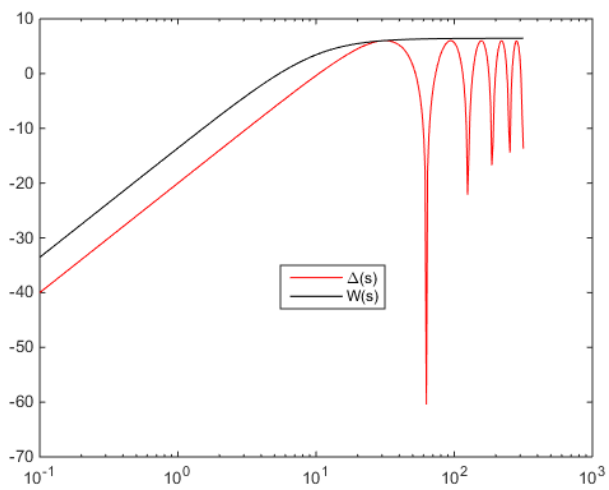


Sensitivity transfer function



Step-response





0.22

