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Distributed Sliding Mode Control for Multi-vehicle Systems with Positive Definite Topologies

Yujia Wu[1], Shengbo Eben Li[2], Yang Zheng[3], and J. Karl Hedrick[1]

***Abstract*— The topological variety significantly affects the platooning of multi-vehicle systems. This paper presents a distributed sliding mode control (SMC) method for vehicular platoons with positive definite topologies. The platoon model is assumed to be homogeneous with strict-feedback nonlinear node dynamics. The design of distributed SMC is divided into two parts, *i.e.*, topological sliding surface design and topological reaching law design. In the former, the sliding surface is defined by weighted summation of individual error, while in the latter, a topologically structured reaching law is proposed to conform with the type of information flow exchange. The Lyapunov method is exploited to prove asymptotic stability of the multi-vehicle system. The effectiveness of this method is validated by numerical simulations.**

I. Introduction

The platooning of multi-vehicle system attracts increasing attentions due to its potential to benefit highway traffic, *e.g.*, improving traffic utility, enhancing driving safety, and reducing fuel consumption [1]. The objective of platoon control is to ensure all the vehicles in a platoon run at a harmonized speed while maintaining the desired inter-vehicle gaps [2], [3].

The earliest platoon control dates back to the well-known PATH project, where linear control strategies were employed for linearized vehicle models in a rigid formation [4]. Since then, many issues on platoon control have been discussed, including control architecture, platoon modeling, spacing policy, controller synthesis, and performance requirements. Nowadays, many researchers have begun to study platoon control from the viewpoint of multi-agent consensus, which is able to further enhance platoon performances in a systematic way [5]. Existing examples include the selection of spacing policies [6], string stability [7], scalability [8], direct consideration of powertrain dynamics [9], dynamic homogeneity and heterogeneity [10], [11]. A recent review on platoon control can be found in [12].

The information flow topology plays a key role to the design of multi-agent consensus based platoon control [12]. Most of earlier literature on platoon control only used radar-based sensing systems, where the type of topologies is quite limited [13]. However, the rapid deployment of vehicle-to-vehicle (V2V) communications, such as DSRC [14], creates the possibility of various topologies. New challenges naturally arise due to this topological variety, in particular when systematically considering node nonlinearity, communication delay and topological switch, *etc*. In such cases, it is more preferable to view the vehicular platoon as a multi-agent system, and to employ a networked control perspective to design distributed controllers [5], [8]. Nowadays, advanced control methods have been introduced to platoon control. For instance, Barooah *et al.* (2009) introduced a mistuning-based control method to improve the stability margin of vehicular platoons [15]. Ploeg *et al.* (2014) developed a H_∞ control method, in which the string stability was explicitly satisfied [16]. A general linear control method for both fixed and switching topologies was discussed from a network viewpoint in [17], and the impaction of connectivity on performance was also analyzed. More recently, some experiments of vehicular platoons have been demonstrated in the real world, including Energy-ITS in Japan [18], SARTRE in Europe [19], and GCDC in the Netherlands [20], *etc.*

The sliding mode control (SMC) is a promising method for platooning of multiple vehicles to handle nonlinear dynamics, actuator constraints, and topological variety. Swaroop and Hedrick (1996) proposed an adaptive SMC for equilibrium-stable interconnected systems, which guaranteed the string stability [7]. In this study, the applied topologies are limited to unidirectional topologies, which means one node can only obtain the information from its predecessors. In [21], a linear SMC was applied to a linearized heterogeneous platoon with time delay and predecessor-following topologies. For the sliding mode design, a posterior tuning or adaptation is required to ensure practical string stability. Also, the SMC was deployed in a predecessor-following topology to cope with communication delay in [22]. String stability is preserved if all vehicles have synchronized information update and the time delay is bounded to a small value. Other examples can be found in [23], [24], and [25]. The main shortcoming of aforementioned SMCs is that they all focused on some specific topologies, for example, unidirectional type in [7], predecessor-following types in [21] and [22], bidirectional types in [26], etc.

This paper presents a distributed sliding mode control method for vehicular platoons with generic topologies, as long as the associated matrices of such topologies are positive definite. Here, the platoon is assumed to be homogeneous with strict-feedback nonlinear node dynamics. The distributed SMC design is divided into two parts, *i.e.*,

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[1] Y. Wu and J. K. Hedrick are with Department of Mechanical Engineering, University of California, Berkeley, CA, 94720. Email: {yujia.wu@, khedrick@me.}berkeley.edu

[2] S. Eben Li is with Department of Automotive Engineering, Tsinghua University, Beijing, 100084, China. Email: lisb04@gmail.com

[3] Yang Zheng is with Department of Engineering Science, University of Oxford, UK. Email: yang.zheng@eng.ox.ac.uk

topological sliding surface design and topological reaching law design. In the former, the sliding surface is designed by multiple sliding surface control method, while in the latter, a topologically structured reaching law is proposed to conform with the type of information flow exchange. Stability is proved by Lyapunov method. The relationship between reaching time of SMC and information topology is discussed analytically. The rest of this paper is organized as follows. The platoon control problem is given in Section II. In Section III, how to model the topology and node dynamics is introduced. Design of distributed SMC is shown in Section IV, followed by a stability proof in Section V, and simulation verification in Section VI. Section VII concludes this paper.

II. Problem Description

A vehicle platoon is a typical multi-agent system, as shown in Fig. 1. As suggested by [5], [8] and [12], a platoon can be decomposed into four main components from the perspective of networked control, *i.e.*, node dynamics, distributed controller, information flow topology, and formation geometry. The node dynamics describe the behavior of each node; the information flow topology defines how nodes exchange information with each other; the distributed controller implements feedback control for each vehicle; and the formation geometry dictates the desired distance between any two successive nodes.

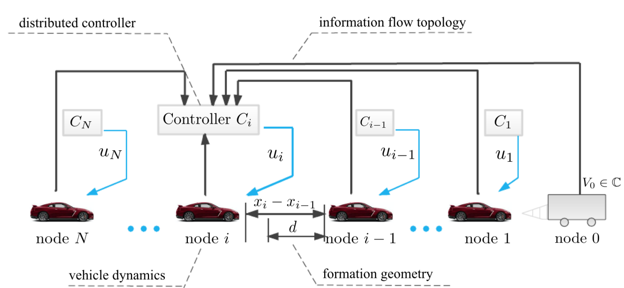

Fig. 1. Four components of a platoon : a) vehicle dynamics, b) information flow topology, c) distributed controller, d) geometry formation [5], [8] and [12]

This platoon contains a leader, denoted by 0, and N followers, denoted by $i \in \mathcal{N} \triangleq \{1, \ldots, N\}$. The leader is assumed to run with a constant speed v_0. The position of the leader is

$$x_0(t) = x_0(0) + v_0 \cdot t, \tag{1}$$

where v_0 is constant and $x_0(0)$ is initial position. In this paper, the desired distance between two neighboring vehicles is denoted to be a constant d. The desired position for each vehicle in the platoon is

$$x_{i,des}(t) = x_0(t) - i \cdot d. \tag{2}$$

where $x_{i,des}$ is the desired position of i-th vehicle. The purpose of platoon control is to ensure all the vehicles to run at a harmonized speed while maintaining the desired inter-vehicle spaces.

III. Platoon Model

A. Model for Information Flow Topology

The information flow topology of a platoon can be modeled by a directed graph $G = \{V, E\}$, in which $V = \{0, 1, \ldots, N\}$ is the node set, and $E \subseteq V \times V$ is the edge set. The following three matrices are used to represent the connectivity in G:

- Adjacent matrix $\mathcal{A}$
- Laplacian matrix $\mathcal{L}$
- Pinning matrix $\mathcal{P}$

The technique that uses matrices to study graphs is known as algebraic graph theory [27], which has recently been used to model the influence of different topologies on platoon performance in [3] and [13]. The adjacent matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ and

$$\begin{cases} a_{ij} = 1, & \{j, i\} \in E, \\ a_{ij} = 0, & \{j, i\} \notin E, \end{cases} \quad i, j \in \mathcal{N}, \tag{3}$$

where $\{j, i\} \in E$ means there is a directional edge from node j to node i, *i.e.*, node i receives the information of j. It is assumed that there are no self-loops, *i.e.*, $a_{ii} = 0$, $i \in \mathcal{N}$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ is then defined as:

$$l_{ij} = \begin{cases} -a_{ij}, & i \neq j, \\ \sum\limits_{k=1,\, k\neq i}^{N} a_{ik}, & i = j, \end{cases} \quad i, j \in \mathcal{N}. \tag{4}$$

The pinning matrix $\mathcal{P}$ represents how each follower connects to the leader, defined as

$$\mathcal{P} = \mathrm{diag}\{p_1, p_2, \ldots, p_N\}, \tag{5}$$

where p_i is used to indicate the existence of edge from leader to node i, *i.e.*, if $p_i = 1$, node i can receive the leader's information; $p_i = 0$, otherwise.

A directed path from node i_1 to node i_k is a sequence of edges (i_1, i_2), (i_2, i_3), $\ldots$, (i_{k-1}, i_k) with all of them in E. For any $(j, i) \in E$, node j is called the parent of node i, node i is called a child of node j. The neighbor set of node i is denoted by $\mathbb{N}_i = \{j | a_{ij} = 1, j \in \mathcal{N}\}$. A tree is a directed graph where a node has no parent and other nodes have exactly one parent. Graph G is said to contain a spanning tree if the tree contains every nodes of the graph. If there is a path between any two nodes of a graph G, then G is connected. We call that the information flow between followers in the graph G is undirected if and only if $a_{ij} = a_{ji}, \forall i, j \in \mathcal{N}$.

Assumption 1: This paper assumes that there exists a spanning tree in G and the information flow between followers is undirected.

Lemma 1: If G satisfies Assumption 1, $\mathcal{L} + \mathcal{P}$ is positive definite.

Proof: When information flow between followers is undirected and connected, $\mathcal{L}$ is positive semi-definite, and the algebraic multiplicity of zero eigenvalue is one. Eigenvector corresponding to zero eigenvalue is $\mathbf{1} \triangleq [1, 1, \ldots, 1]^\top \in \mathbb{R}^N$ [27]. Define eigenvalues of $\mathcal{L}$ to be $\lambda_1 = 0 < \lambda_2 < \ldots < \lambda_N$, and corresponding eigenvectors are

$\eta_1, \eta_2, \ldots, \eta_N$, where $\eta_1 = \mathbf{1}$. The whole space $\mathbb{R}^N$ is the composition of eigenspace and nullspace, so any vectors $x \in \mathbb{R}^N$ could be written as a linear composition of eigenvectors, $x = \sum_{i=1}^{N} c_i \eta_i$, where c_i, $i \in \mathcal{N}$ are constants.

Since G contains a spanning tree, $\mathcal{P} \neq 0$, and $\eta_1^\top \mathcal{P} \eta_1 > 0$. For any $x \neq 0$, there is

$$x^\top (\mathcal{L} + \mathcal{P}) x = \sum_{i=2}^{N} \lambda_i c_i^2 \eta_i^\top \eta_i + x^\top \mathcal{P} x > 0. \tag{6}$$

□

Remark 1: Information flow topology is said to be positive definite if $\mathcal{L} + \mathcal{P}$ is positive definite. Similar proof of Lemma 1 could be found in [28].

B. Nonlinear Model for Node Dynamics

The vehicle longitudinal dynamics are nonlinear systems, which are composed of engine, drive line, brake systems, aerodynamics drag, tire friction, rolling resistance, gravitational forces, *etc*. To strike a balance between accuracy and conciseness, it is assumed that: (1) the vehicle body is rigid and left-right symmetric; (2) the platoon is on flat and dry-asphalt road, and the tire slip in the longitudinal direction is neglected; (3) the powertrain dynamics are lumped to be a first-order inertial transfer function; (4) the driving and braking torques are integrated into one control input [29][30]. For a homogeneous vehicle platoon, the i-th node dynamics are described as

$$\dot{x}_i(t) = v_i(t), \tag{7}$$

$$\dot{v}_i(t) = \frac{1}{m}\left(\eta_T \frac{T_i(t)}{R} - C_A v_i^2(t) - mgf\right), \tag{8}$$

$$\dot{T}_i(t) = \frac{u_i(t) - T_i(t)}{\tau}, \tag{9}$$

where $x_i(t)$, $v_i(t)$, $T_i(t)$ are distance, velocity and drive torque; $u_i(t)$ represents the desired driving/braking torque; m is the mass of vehicle; η_T is the mechanical efficiency of the driveline; R is radius of wheel; C_A is the coefficient of aerodynamic drag; g is gravitational constant; f is the coefficient of rolling resistance; and τ is inertial lag of longitudinal dynamics.

To design a distributed SMC, the tracking problem described by section II needs to be transformed to a regulation problem. In this paper, the leader's velocity is assumed to be a constant value, which is a standard assumption in the literature [3], [12], [13], [15]. Then, we further assume the velocity of the leader can be broadcasted to all nodes via multi-hopping, since there exists a spanning tree rooting from the leader in graph G.

The equilibrium of each node is calculated by

$$v_{eq} = v_0, \tag{10}$$

$$T_{eq} = \frac{R}{\eta_T}(C_A v_{eq}^2 + mgf). \tag{11}$$

Then by defining

$$\Delta x_i(t) \triangleq x_i(t) - v_{eq} \cdot t, \tag{12}$$

$$\Delta v_i(t) \triangleq v_i(t) - v_{eq}, \tag{13}$$

$$\Delta T_i(t) \triangleq T_i(t) - T_{eq}, \tag{14}$$

the tracking was converted to a regulation problem with the control objective $\Delta x_i(t) \to (i \cdot d - x_0(0))$.

After removing the equilibrium, the dynamics becomes

$$\Delta \dot{x}_i(t) = \Delta v_i(t), \tag{15}$$

$$\Delta \dot{v}_i(t) = \frac{1}{m}(\eta_T \frac{\Delta T_i(t) + T_{eq}}{R} - C_A(\Delta v_i(t) + v_{eq})^2 - mgf), \tag{16}$$

$$\Delta \dot{T}_i(t) = \frac{u_i(t) - \Delta T_i(t) - T_{eq}}{\tau}. \tag{17}$$

IV. Distributed SMC for Nonlinear Platoon

The distributed SMC design is divided into two parts, *i.e.*, topological sliding surface design and topological reaching law design. In the topological sliding surface design, the dynamics of each node falls into the category of strict-feedback systems. Synthetic control techniques, such as multiple sliding surface control, backstepping control and dynamic surface control, can be used to design the sliding surface. In this paper, the multiple sliding surface control is firstly applied to each individual node dynamics, (15)-(17), to generate desired torque, which is a synthetic control. Then, the errors between each ΔT_i and $\Delta T_{i,des}$, which are defined as intermediate errors, are used to construct the topological sliding surface. In the reaching law design, a topologically structured reaching law is proposed by using the elements of $\mathcal{L} + \mathcal{P}$ as the weighting coefficients for sliding errors. Such design leads to a distributed SMC which implements physical control only using the information from neighbor set.

A. Design of sliding surface via multiple surface control

For the longitudinal tracking task of a single vehicle, three-layer multiple sliding surface control design (i.e., position-layer, velocity-layer, and torque-layer) is often used to derive its control law. In this paper, only first two layers are used, and the third layer is replaced by distributed sliding surface constructed from torque tracking errors. The distributed sliding surface is the summation of torque tracking errors from neighboring nodes weighted by the elements of $\mathcal{L} + \mathcal{P}$.

1) First layer design: For the first layer of (15), $\Delta v_{i,des}$ is regarded as synthetic control input for position tracking. With desired position defined as

$$\Delta x_{i,des} \triangleq i \cdot d - x_0(0). \tag{18}$$

The position error is defined as,

$$e_{i,1}(t) \triangleq \Delta x_i(t) - \Delta x_{i,des}(t). \tag{19}$$

By Lyapunov design,

$$\dot{e}_{i,1}(t) = -\lambda_1 e_{i,1}(t), \tag{20}$$

where λ_1 is a tuning parameter denoting the converging rate of tracking error. Substituting (18), (15) and (19) to (20), we get

$$\Delta v_{i,des}(t) = -\lambda_1 e_{i,1}(t), \tag{21}$$

where $\Delta v_{i,des}$ is the synthetic control. Eq. (21) means that if Δv_i is equal to the right hand side of the equation, tracking objective can be achieved.

2) Second layer design: For the second layer, the goal is to let Δv_i track $\Delta v_{i,des}$. According to dynamics (16), synthetic control is $\Delta T_{i,des}$. The velocity tracking error is defined as

$$e_{i,2}(t) \triangleq \Delta v_i(t) - \Delta v_{i,des}(t). \tag{22}$$

By Lyapunov design,

$$\dot{e}_{i,2}(t) = -\lambda_2 e_{i,2}(t), \tag{23}$$

where λ_2 is also a tuning parameter denoting the converging rate of velocity error. Substituting (16) and (22) to (23), desired torque can be obtained

$$\begin{aligned}\Delta T_{i,des}(t) =& \frac{R}{\eta_T}(mfg + C_A(v_{eq} + \Delta v_i(t))^2 \\ &- m\lambda_1 \Delta v_i - m\lambda_2 e_{i,2}(t)) - T_{eq}.\end{aligned} \tag{24}$$

3) Design of distributed topological sliding surface: Define the error between actual torque and desired torque as

$$\Delta_i(t) \triangleq \Delta T_i(t) - \Delta T_{i,des}(t). \tag{25}$$

The individual sliding error is defined as the weighted summation of Δ_i, for $i = 1, \ldots, N$,

$$s_i(t) \triangleq \sum_{j=1, j\neq i}^{N} a_{ij}(\Delta_i(t) - \Delta_j(t)) + p_i \Delta_i(t), \tag{26}$$

where a_{ij} and p_i are elements from adjacent matrix and pining matrix. The sliding surface of the whole system is topologically structured by $\mathcal{L} + \mathcal{P}$,

$$S(t) = \begin{bmatrix} s_1(t) \\ s_2(t) \\ \vdots \\ s_N(t) \end{bmatrix} = (\mathcal{L} + \mathcal{P}) \begin{bmatrix} \Delta_1(t) \\ \Delta_2(t) \\ \vdots \\ \Delta_N(t) \end{bmatrix}. \tag{27}$$

Remark 2: Each individual sliding error $s_i(t)$ only contains node states allowed by self-measurement and communication among neighboring nodes because of the use of a_{ij} and p_i as weighting coefficients, which means that $s_i(t)$ is designed in a locally distributed way. Note that $\Delta_i(t) - \Delta_j(t)$ does not contain any information from the leader since

$$\begin{aligned}\Delta_i(t) - \Delta_j(t) =& \Delta T_i(t) - \Delta T_j(t) \\ &+ \frac{R}{\eta_T}(\lambda_1 \lambda_2 m(\Delta x_i(t) - \Delta x_j(t) \\ &- (i-j)d)) \\ &- \frac{R}{\eta_T} C_A(\Delta v_i - \Delta v_j)(\Delta v_i + \Delta v_j \\ &+ 2v_{eq} - m\lambda_2)\end{aligned} \tag{28}$$

Remark 3: The collective sliding error $S(t)$ then becomes a linear transform of torque error vector related to $\mathcal{L} + \mathcal{P}$ in (27). Because $\mathcal{L} + \mathcal{P}$ is invertible according to Assumption 1, this linear transform is one-to-one unique mapping.

B. Design of topologically structured reaching law

To obtain distributed SMC, which only uses the states from self-node and neighboring nodes, the reaching law have to conform with afore-designed collective sliding error. A similar weighting fashion is adopted in this paper, which yields a newly proposed topologically structured reaching law.

The individual topological reaching law is designed as

$$\begin{aligned}\dot{s}_i(t) =& -\psi(\sum_{j=1, j\neq i}^{N} a_{ij}(s_i(t) - s_j(t)) + p_i s_i(t)) \\ &- \phi(\sum_{j=1, j\neq i}^{N} a_{ij}(\mathrm{sgn}(s_i(t)) - \mathrm{sgn}(s_j(t))) \\ &+ p_i\, \mathrm{sgn}(s_i(t))),\end{aligned} \tag{29}$$

where $\psi > 0$ and $\phi > 0$ are tuning parameters.

Write in array form, we obtain the collective topological reaching law

$$\begin{aligned}\dot{S}(t) =& \begin{bmatrix} \dot{s}_1(t) \\ \dot{s}(t) \\ \vdots \\ \dot{s}_N(t) \end{bmatrix} \\ =& -(\mathcal{L} + \mathcal{P})(\psi S(t) + \phi\, \mathrm{sgn}(S(t))),\end{aligned} \tag{30}$$

where $\mathrm{sgn}(S(t)) \triangleq [\mathrm{sgn}(s_1(t)), \ldots, \mathrm{sgn}(s_N(t))]^\top \in \mathbb{R}^N$.

Compare the derivative of (27) and the designed reaching law (30),

$$(\mathcal{L} + \mathcal{P}) \begin{bmatrix} \dot{\Delta}_1(t) \\ \dot{\Delta}_2(t) \\ \vdots \\ \dot{\Delta}_N(t) \end{bmatrix} = -(\mathcal{L} + \mathcal{P})(\psi S(t) + \phi\, \mathrm{sgn}(S(t))). \tag{31}$$

Since $\mathcal{L} + \mathcal{P}$ is invertible, $\mathcal{L} + \mathcal{P}$ can be canceled:

$$\begin{bmatrix} \dot{\Delta}_1(t) \\ \dot{\Delta}_2(t) \\ \vdots \\ \dot{\Delta}_N(t) \end{bmatrix} = -(\psi S(t) + \phi\, \mathrm{sgn}(S(t))). \tag{32}$$

The cancellation of $\mathcal{L} + \mathcal{P}$ is the key stone of designing distributed SMC for a broad range of topologies. By comparing each term in the array, each node corresponds to an equation:

$$\dot{\Delta}_i(t) = -\psi s_i(t) - \phi\, \mathrm{sgn}(s_i(t)). \tag{33}$$

Substituting (17) and (25) into (33), the control input is equal to

$$\begin{aligned}u_i(t) =& \tau(\Delta \dot{T}_{i,des}(t) - \psi s_i(t) - \phi\, \mathrm{sgn}(s_i(t))) \\ &+ \Delta T_i(t) + T_{eq}.\end{aligned} \tag{34}$$

To write (34) in an explicit control law, substituting (24) to (34) yields

$$\begin{aligned} u_i(t) =& \Delta T_i(t) + T_{eq} - \tau(\psi s_i(t) + \phi \operatorname{sgn}(s_i(t))) \\ &+ \tau \frac{R}{\eta_T}(2C_A \Delta \dot{v}_i(t)(\Delta v_i(t) + v_{eq}) \\ &- m\lambda_1 \Delta \dot{v}_i - m\lambda_2(\Delta \dot{v}_i(t) + \lambda_1 \Delta v_i(t))). \end{aligned} \tag{35}$$

The control law (35) is distributed for each node in the sense that its feedback only uses the states from self-node, neighboring nodes and the leader if only pinning to the leader.

V. Stability Proof

The stability proof of distributed SMC is also divided into two phases, i.e., reaching phase and sliding phase. The stability of reaching phase is analyzed by Lyapunov method, while that of sliding phase follows the procedure of multiple sliding surface stability analysis.

A. Reaching Phase

Theorem 1: Consider a platoon with nonlinear node dynamics described by (15)-(17) and topologies under Assumption 1. With the distributed controller (35) and tuning parameters $\phi > 0$ and $\psi > 0$, then sliding surface $S(t) = \mathbf{0}$ could be reached in a finite time bounded by $\frac{\|S(0)\|_2}{\phi \sigma_{\min}}$, where $\sigma_{\min}$ is smallest eigenvalue of $\mathcal{L} + \mathcal{P}$.

Proof: Choose Lyapunov candidate for the networked system

$$V(t) = \frac{1}{2} S(t)^\top S(t). \tag{36}$$

Taking the derivative of Lyapunov function,

$$\begin{aligned} \dot{V}(t) =& S^\top(t)\dot{S}(t) \\ =& - S^\top(t)(\mathcal{L} + \mathcal{P})(\psi S(t) + \phi \operatorname{sgn}(S(t))). \end{aligned} \tag{37}$$

By the definition of $\operatorname{sgn}(S(t))$, there is

$$\begin{aligned} \operatorname{sgn}(S(t)) =& [\frac{s_1(t)}{|s_1(t)|}, \dots, \frac{s_N(t)}{|s_N(t)|}]^\top \\ \geq& \frac{S(t)}{\|S(t)\|_2} \end{aligned} \tag{38}$$

From (37), we get

$$\begin{aligned} \dot{V}(t) =& - S^\top(t)(\mathcal{L} + \mathcal{P})(\psi S(t) + \phi \operatorname{sgn}(S(t))) \\ \leq& - \phi S^\top(t)(\mathcal{L} + \mathcal{P}) \operatorname{sgn}(S(t)) \\ \leq& \frac{-\phi S^\top(t)(\mathcal{L} + \mathcal{P})S(t)}{\|S(t)\|_2} \end{aligned} \tag{39}$$

Apply Rayleigh's Quotient to (39)

$$\begin{aligned} \dot{V}(t) \leq& \frac{-\phi S^\top(t)(\mathcal{L} + \mathcal{P})S(t)}{\|S(t)\|_2} \\ \leq& -\phi \sigma_{\min} \|S(t)\|_2, \end{aligned} \tag{40}$$

where $\sigma_{\min} > 0$ is the smallest eigenvalue of $\mathcal{L} + \mathcal{P}$.

Lyapunov candidate (36) could also be rewritten as

$$V(t) = \frac{1}{2} \|S(t)\|_2^2. \tag{41}$$

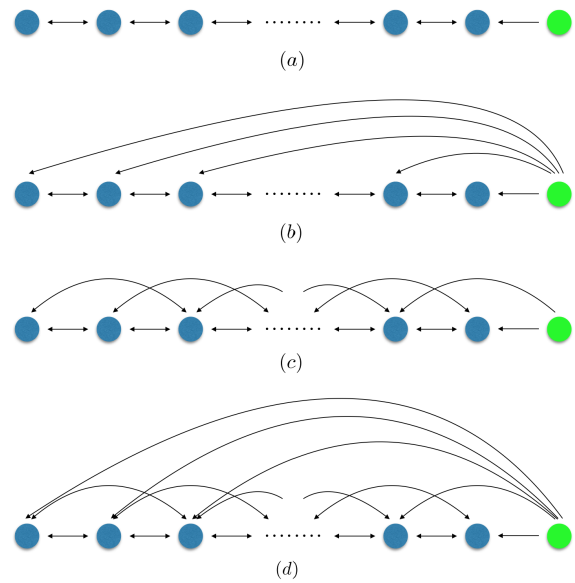

Fig. 2. Typical types of information flow topology [8]: (a) bidirectional type; (b) bidirectional-leader type; (c) symmetric-double-nearest-neighbor type; (d) symmetric-double-nearest-neighbor leader type.

The derivative of Lyapunov function is

$$\dot{V}(t) = \|S(t)\|_2 \frac{d}{dt}(\|S(t)\|_2). \tag{42}$$

Compare (40) to (42), we get

$$\frac{d}{dt}(\|S(t)\|_2) \leq -\phi \sigma_{\min} \tag{43}$$

From (40) and (43), one can conclude that the Lyapunov function holds, and reaching time t_r for $\|S(t_r)\|_2 = 0$ is bounded by $t_r \leq \frac{\|S(0)\|_2}{\phi \sigma_{\min}}$.

Remark 4: The upper bound of reaching time is largely affected by the minimum eigenvalue of matrix $\mathcal{L} + \mathcal{P}$ associated with the topology, which also hints us to choose a topology with a larger minimum eigenvalue to reduce the reaching time. Interestingly, this result agrees with some recent findings on stability margin analysis [3], [8], [31], where the minimum eigenvalue also exerts greatly influences on the scaling trend of stability margin.

□

B. Sliding Phase

Theorem 2: Consider a vehicle platoon with nonlinear dynamics described by (15)-(17) and topologies under Assumption 1. During the sliding phase where $S(t) = \mathbf{0}$, the platoon is asymptotically stable.

Proof: Since $\mathcal{L} + \mathcal{P}$ is positive definite, (27) shows

$$S = (\mathcal{L} + \mathcal{P}) \begin{bmatrix} \Delta_1(t) \\ \Delta_2(t) \\ \vdots \\ \Delta_N(t) \end{bmatrix} = \mathbf{0}, \tag{44}$$

thus,

$$\Delta_i(t) = 0, \quad \forall i \in \mathcal{N}. \tag{45}$$

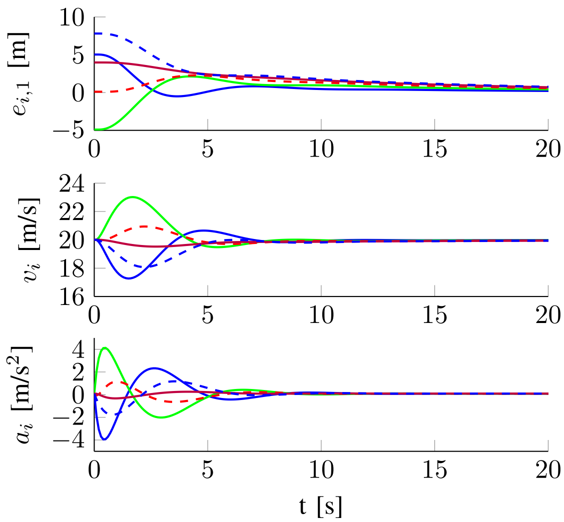

Fig. 3. Case (a): bidirectional topology

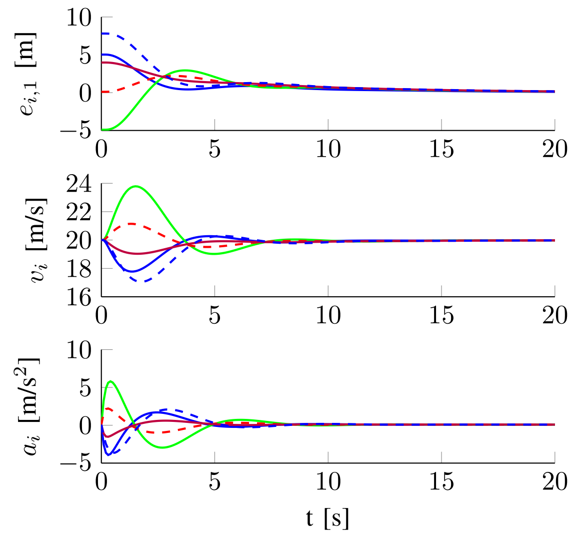

Fig. 5. Case (c):symmetric-double-nearest-neighbor topology

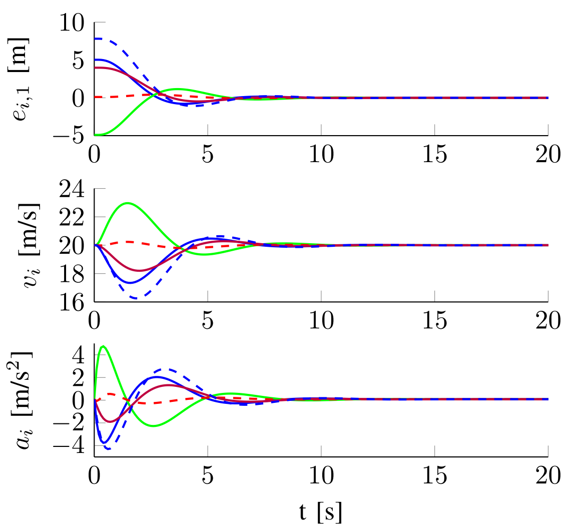

Fig. 4. Case (b): bidirectional-leader topology

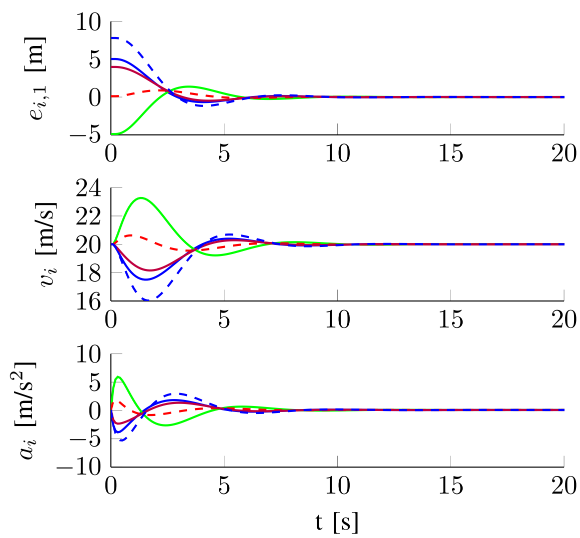

Fig. 6. Case (d):; symmetric-double-nearest-neighbor leader topology

The former equation (45) implies $\Delta T_i(t) = \Delta T_{i,des}(t)$. Choose a Lyapunov candidate function for each vehicle

$$V_i = \frac{e_{i,1}^2(t) + e_{i,2}^2(t)}{2}. \tag{46}$$

Take derivative of Lyapunov candidate, we get

$$\begin{aligned}\dot{V}_i =& e_{i,1}(t)\dot{e}_{i,1}(t) + e_{i,2}(t)\dot{e}_{i,2}(t) \\ =& -\lambda_1 e_{i,1}^2(t) - \lambda_2 e_{i,2}^2(t) + e_{i,1}(t)e_{i,2}(t).\end{aligned} \tag{47}$$

If choose $\lambda_1 > 0.5$, $\lambda_2 > 0.5$, the derivative $\dot{V}_{2,i}$ is negative definite. □

VI. Simulation Results

The effectiveness of proposed distributed SMC is illustrated by numerical simulations in this section. A homogeneous platoon with one leader and five followers is simulated under four different topologies. Specifically, as shown in Fig. 2, we consider 1) bidirectional, 2) bidirectional-leader, 3) symmetric-double-nearest-neighbor, and 4) symmetric-double-nearest-neighbor leader topology [8].

As stated in section II, the leader vehicle is assumed to be driving with a constant speed $v_0 = 20$ m/s. The initial position of leader was set to $x_0(0) = 10$ m. Some key vehicle parameters are listed: mass of vehicle, $m = 1645$ kg; powertrain mechanical efficiency, $\eta_T = 0.76$; wheel radius, $R = 0.3$ m; coefficient of aerodynamic drag, $C_A = 0.3$; gravitational constant, $g = 9.8$ m/s^2; coefficient of rolling resistance, $f = 0.02$, and inertial lag of powertrain dynamics, $\tau = 0.3$ s. The desired inter-vehicle distances are all set to be 10 m. The initial inter-vehicle distances are randomly set between $0 - 20$ m. Simulation parameters are chosen that $\lambda_1 = 1$, $\lambda_2 = 1$, $\psi = 1$, and $\phi = 1$.

Simulation results for four different topologies are shown by Fig. 3 - Fig. 6. Each figure contains 3 subplots, i.e., position error $e_{i,1}$, speed of each vehicle v_i, and acceleration of each vehicle a_i. The 1st, 2nd, 3rd, 4th and 5th vehicle are denoted by (——), (——), (- - -), (——), (- - -), respectively.

We observe that asymptotic stability are achieved for all four topologies. Moreover, for the case (b) and (d), for which the smallest eigenvalues of $\mathcal{L}+\mathcal{P}$ are both 1, the converging speed is faster than case (a) and (b), for which the smallest eigenvalues of $\mathcal{L}+\mathcal{P}$ are 0.08 and 0.14, respectively, which conforms to our discussions in Remark 4.

VII. Conclusion

The rapid deployment of vehicle-to-vehicle (V2V) communications generates a variety of topological types for platoon control. This paper proposed a distributed sliding mode control (SMC) method for homogeneous vehicular platoons with nonlinear dynamics and positive definite topologies. The distributed SMC design is divided into two parts, *i.e.*, topological sliding surface design and topological reaching law design. In the former, the sliding surface is designed by multiple sliding surface control method, while in the latter, a topologically structured reaching law is proposed to conform with the type of information flow exchange. The asymptotic stability is proved by Lyapunov method, which also shows that the minimum eigenvalue of $\mathcal{L}+\mathcal{P}$ has a large impact to reaching time of distributed SMC. Numerical simulations demonstrated its effectiveness to handle nonlinear node dynamics and different types of topologies.

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