Behavioral Cooperation of Multiple Connected Vehicles with Directed Acyclic Interactions using Feedforward-Feedback Control

Yougang Bian[1], Yang Zheng[1], Shengbo Eben Li[1], Qing Xu[1], Jianqiang Wang[1,2], Keqiang Li[1]

[1] Department of Automotive Engineering, Tsinghua University, Beijing, China
[2] Collaborative Innovation Center of Electric Vehicles in Beijing, Beijing, China

Corresponding author: Keqiang Li; E-mail: likq@tsinghua.edu.cn

This paper presents a behavioral cooperation method for multiple connected vehicles with directed acyclic interactions. Our main idea is based on a novel feedforward-feedback control scheme, in which each vehicle takes the average of neighbors' control inputs as a feedforward term and the average of neighbors' state errors as a feedback term. In this way, the control inputs are mutually dependent in a group of connected vehicles. We show taht the control inputs can be calculated sequentially according to the topological ordering of the directed acyclic communication topology. Further, we prove that the proposed controller guarantees the asymptotic stability and is optimal with respect to a specific quadratic performance index, since it implicitly uses the leader's information. Simulation results demonstrate the advantage of the proposed control method.

Topics / Connected vehicles, Platoon, Cooperative control, Distributed control

1. INTRODUCTION

Road safety and fuel efficiency are two eternal themes for transportation systems. Nowadays, the behavioral cooperation of multiple connected vehicles has become an effective approach to meet these demands. In the one-dimensional case, the cooperation is referred to as the *vehicle platooning* or *cooperative adaptive cruise control* (CACC), of which the earliest research can be dated back to the well-known PATH program [1]. Up till now, many demonstrations of vehicle platoons have already been conducted in real world, including the Safe Road Trains for the Environment (SARTRE) [2] project in Europe, the Grand Cooperative Driving Challenge (GCDC) competition [3] in the Netherlands, and the Energy-ITS project [4] in Japan. An overview of recent advances in platoon control techniques can be found in [5].

In a platoon, multiple vehicles are coordinated to move at the same speed while maintaining a desired inter-vehicle distance [6]. One direct benefit is that thanks to cooperation among vehicles, the inter-vehicle distance may be decreased so as to reduce the aerodynamic drag, leading to a certain improvement on fuel economy [7]. Many advanced control methods, *e.g.*, distributed sliding mode control (SMC) [8][9], distributed receding horizon control [10][11], distributed $\mathcal{H}_\infty$ control [12]-[14] and distributed periodic control [15][16], have been applied to achieve the global stability of platoons with desirable longitudinal control performance. The notion of string stability, *i.e.*, the ability to attenuate the propagation of error along the platoon, plays a key role to guarantee the safety. In the literature, different range policies are discussed to achieve the string stability [17]. As demonstrated by [18], compared with adaptive cruise control (ACC) systems, cooperative ACC systems have the potential to further improve highway capacity and traffic flow stability. Even in a mixed traffic flow, where conventional, ACC, and communication-assisted vehicles exist at the same time, a similar technique called connected cruise control (CCC) can also help to maintain the smoothness of traffic flow [19][20]. The degradation from CACC to ACC when communication faults occur is also studied to partially maintain the string stability [21].

In the early stage, onboard sensors, such as radars, were used in platoons for environment perception, so vehicles could only use their own relative measurements for platoon control. Nowadays, V2V communication is applied to vehicle platoons for performance enhancement. Two main impacts of introducing communication to platoons are:

First, communication makes it possible to transmit more information, such as the absolute acceleration and the control input, among vehicles for feedforward control design. Commonly used feedforward strategies include the input signal feedforward [6][13][18][21], acceleration feedforward [22], and predicted acceleration feedforward [23]. For example, an input signal feedforward-based $\mathcal{H}_\infty$ controller synthesis approach was proposed in [13] for CACC systems with predecessor following (PF) and two-predecessor following (TPF) topologies to achieve string stability. In [18], an input signal feedforward control strategy was also discussed for predecessor-leader following (PLF) communication topology. An acceleration feedforward controller was designed in [22] for heterogeneous vehicle platoons with PF topology, where a necessary and sufficient frequency-domain condition for string stability was derived. A comparison study of the three types of feedforward strategies for vehicle platoons with PF topology is given in [23], where the range of vehicle model and controller parameters for string stability was numerically computed using a bisection method.

Second, V2V communication brings various information topologies, for which the modeling and analyzing strategy remains a challenging topic. Fig. 1 lists some commonly used topologies in vehicle platoons. The above-mentioned studies only considered specific communication topologies: for instance, [13] only considered the PF and TPF topologies, [18] only considered the PLF topology, and [22][23] only considered the PF topology, which may restrict the application range of the proposed control methods. A promising way to address this issue is to use consensus-based methods [24], where the communication topology is characterized by algebraic graph theory to systematically study its effect on the whole system. For example, a separation principle was proposed in [25] to decompose the stability of a vehicle formation into two components: the stability of the information flow and the stability of individual vehicles, which highlights the significance of the communication topology. In [26], the consensus of multi-agent systems was cast into the stability of a set of low-dimension matrices to reduce the complexity of system analysis and synthesis. This method was further extended to the four-component framework for vehicle platoon control in [27], where a unified internal stability theorem was proved by using the algebraic graph theory and Routh–Hurwitz stability criterion. However, most of the current research on consensus-based platoon control mainly focuses on feedback control, which fails to take full use of V2V communication for feedforward design.

In this paper, we propose a novel behavioral cooperation method that combines feedforward and feedback control together for multiple connected vehicle systems with directed acyclic interactions, which is also considered in our previous work [28]. The main contributions of this paper include: 1) the proposed method combines both feedforward and feedback in the consensus-based platoon control. Compared with previous studies, *e.g.*, [25]-[27], which only consider feedback control, or [13][18][21]-[23], which only take into account specific communication topologies in feedforward control, our method builds a feedforward-feedback framework for consensus-based platoon control that works for a large class of communication topologies; 2) the asymptotic stability and the optimality of the feedforward-feedback control are analyzed, and we prove that the proposed controller is optimal with respect to a specific quadratic performance index. This makes it possible for the explicit performance optimization in the platoon control design.

The rest of the paper is organized as follows. Section 2 presents the system modeling and platoon control objective. Section 3 designs the distributed feedforward-feedback controller and proves the stability and optimality. Numerical simulation results are given in Section 4 and conclusions are drawn in Section 5.

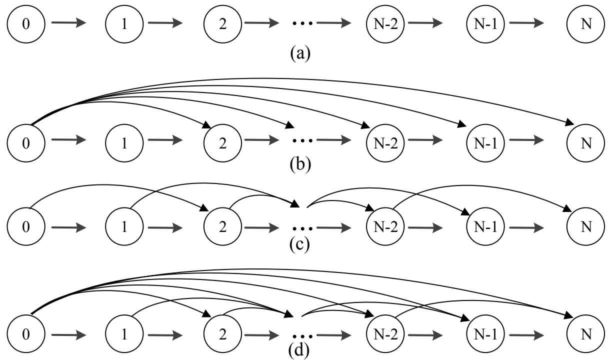


Fig. 1 Commonly used communication topologies [27]. (a) predecessor following (PF), (b) predecessor-leader following (PLF), (c) two-predecessor following (TPF), (d) two-predecessor-leader following (TPLF).

Notations: The fields of real numbers and $m \times n$ real matrices are denoted by $\mathbb{R}$ and $\mathbb{R}^{m\times n}$, respectively. A matrix $M \in \mathbb{R}^{m\times n}$ is represented by its entry m_{ij}, *i.e.*, $M = [m_{ij}]$, and its transpose is denoted by M^T. An $n \times n$ diagonal matrix with entries $m_1, m_2, .., m_n$ starting from the upper left is denoted by $\text{diag}\{m_1, m_2, .., m_n\}$ for convenience. The $n \times n$ identical matrix is denoted by I_n. A time-varying signal $x(t)$ is denoted by x for convenience.

2. PROBLEM STATEMENT

Considers a homogeneous platoon consisting of a leading vehicle indexed by 0 and multiple following vehicles indexed by $1,2, \dots, N$, respectively. The details of the system modeling and control objective are given in the following subsections.

2.1 Model of Vehicle Dynamics

By neglecting the lateral vehicular motions, we consider the following longitudinal vehicle dynamics:

$$\dot{x}_i = Ax_i + Bu_i, i = 0,1,2, .., N, \tag{1}$$

where

$$x_i = \begin{bmatrix} p_i \\ v_i \\ a_i \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{1}{\tau} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\tau} \end{bmatrix}, \tag{2}$$

p_i, v_i, a_i and u_i denote the position, velocity, acceleration and control input (desired acceleration) of vehicle i, respectively; τ represents the inertial time lag in the driveline.

Note that the model (1) assumes that each vehicle is equipped with a low-level acceleration controller that regulates a_i according to u_i. In addition, the dynamics of the low-level acceleration controller can be modeled as a first-order lag system with the time constant τ. That this model is widely used in the literature, *e.g.*, [17][21][27].

2.2 Model of Communication Topology

The communication topology among the following vehicles is modeled with a directed graph denoted by $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$, where $\mathcal{V} = \{V_1, V_2, \dots, V_N\}$ is the set of vertices (or vehicles), $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the adjacency matrix, which characterizes the interactions among the following vehicles. The entries of $\mathcal{A}$ are defined as:

$$a_{ij} = \begin{cases} 1 & \text{if } \{V_j, V_i\} \in \mathcal{E} \\ 0 & \text{otherwise} \end{cases}, \tag{3}$$

where $\{V_j, V_i\} \in \mathcal{E}$ means vehicle i can receive the information from vehicle j. We assume that there is no self-loop, *i.e.*, $a_{ii} = 0, \forall i = 1,2,\ldots,N$.

The extended graph which incorporates the leading vehicle is denoted by $\bar{\mathcal{G}}$. The interactions between the leading vehicle and the following vehicles are modeled with the pinning matrix $\mathcal{P} = \mathrm{diag}\{b_1, b_2, \ldots, b_N\} \in \mathbb{R}^{N\times N}$, where b_i equals 1 if vehicle i can acquire the information of the leading vehicle, or 0 otherwise.

For each following vehicle, we define three sets to represent the neighbor relationship:

(a) A neighbor set $\mathbb{N}_i = \{j | a_{ij} = 1, \forall j \in \mathcal{V}\}$.

(b) A leader accessibility set

$$\mathbb{P}_i = \begin{cases} \{0\} & \text{if } p_{ii} = 1 \\ \emptyset & \text{otherwise} \end{cases}. \tag{4}$$

(c) The union of the above two sets $\mathbb{I}_i = \mathbb{N}_i \cup \mathbb{P}_i$, which contains all the information sources of vehicle i.

In this study, we focus on a specific type of communication topologies by assuming that the graph $\mathcal{G}$ is a directed acyclic graph (DAG), *i.e.*, a finite directed graph with no directed cycles. In addition, we assume that $\bar{\mathcal{G}}$ contains a spanning tree rooted at the leading vehicle, which means that there exists a directed path from the leading vehicle to each following vehicle. For this type of topologies, we have the following lemma.

Lemma 1: For a DAG with a spanning tree, there exists at least one topological ordering, *i.e.*, a sequence of the vertices such that every edge is directed from upstream to downstream in the sequence, starting from the root of the spanning tree.

The proof of this lemma is straightforward, so we omit it here. To illustrate ***Lemma 1***, some examples are given in Fig. 2: graph (a) is a DAG with a spanning tree rooted at vertex 0 while graph (b) is not, since vertices {0, 1, 2} and {0, 3, 2} form two directed cycles. In addition, graphs (c) and (d) are equivalent to graph (a), but the sequences of vertices are shifted so that each edge is directed from upstream (left) to downstream (right) in the sequences. Then, we refer to {0, 3, 1, 2} in (c) and {0, 1, 3, 2} in (d) as two topological orderings of {0, 1, 2, 3} in (a). Besides, both these two topological orderings start from vertex 0, i.e., the root of the spanning tree. Also, note that all the communication topologies in Fig. 1 are DAGs with a spanning tree rooted at the leading vehicle.

2.3 Objective of Platoon Control

The control objective is to keep the desired platoon velocity while maintaining the desired inter-vehicle distance. For all the following vehicles, the tracking error with respect to the leading vehicle is defined as:

$$\tilde{x}_i = \begin{bmatrix} \tilde{p}_i \\ \tilde{v}_i \\ \tilde{a}_i \end{bmatrix} = \begin{bmatrix} p_i - p_0 - d_{i,0} \\ v_i - v_0 \\ a_i - a_0 \end{bmatrix}, i = 1,2,\ldots,N, \tag{5}$$

where $d_{i,0}$ is the desired inter-vehicle distance between vehicle i and 0. Here, we use the constant spacing policy (CSP) [12][27], *i.e.*, $d_{i,0} = i \times d_0$, where d_0 is the standstill gap. Then, the control objective becomes:

$$\lim_{t\to+\infty} \tilde{x}_i(t) = 0, i = 1,2,\ldots,N. \tag{6}$$

Note that in (5), the values of $\tilde{p}_i$, $\tilde{v}_i$ and $\tilde{a}_i$ are available for vehicle i only if the leading vehicle is accessible for vehicle i.

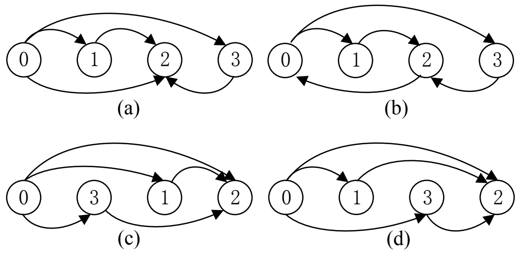


Fig. 2 Examples of DAGs and topological orderings: graphs (a), (c) and (d) are DAGs but graph (b) is not.

3. CONTROLLER DESIGN

In this paper, we assume that the communication between vehicles is perfect without time delay and package loss, as considered in [11][12][14] and [27]. Then, we propose a new feedforward-feedback control scheme as follows:

$$u_i = \underbrace{K_{\mathrm{ff},i}\frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i} u_j}_{\text{feedforward}} - \underbrace{K_{\mathrm{fb},i}\frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}(\tilde{x}_i - \tilde{x}_j)}_{\text{feedback}}, \tag{7}$$

where $|\mathbb{I}_i|$ is the cardinality (number of elements) of $\mathbb{I}_i$; $K_{\mathrm{ff},i}$ and $K_{\mathrm{fb},i}$ denote the feedforward and feedback gain, respectively.

In (7), the feedforward term $\frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i} u_j$ is the average of neighbors' control inputs, and the feedback term $\frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}(\tilde{x}_i - \tilde{x}_j)$ is the average of neighbors' relative state errors. We note that the calculation of $\tilde{x}_i - \tilde{x}_j$ does not require the leader's information. Compared with [12][14][27], which only take $\sum_{j\in\mathbb{I}_i}(\tilde{x}_i - \tilde{x}_j)$ for feedback, the control law (7) makes full use of neighbors' control inputs for feedforward. Compared with [6][18][21][13], which only take u_0, u_{i-1} or $\sum_{j=1}^{k} u_{i-j}$ ($1 \le k \le i$) for feedforward, the control law (7) generalizes the feedforward design by considering general communication topologies.

Remark 1: In (7), vehicles' control inputs are mutually dependent. Here we note that (7) can be calculated sequentially according to the topological ordering of the DAG given in ***Lemma 1***, since the downstream vehicles in the ordering only need the control inputs of the upstream ones. This idea is similar to that in [11], where the control inputs of vehicles can be calculated using the distributed model predictive control (DMPC) according to the ordering of vehicles in the unidirectional communication topology.

Remark 2: If there exists time delay in the calculation of control inputs, then the vehicle i cannot obtain its neighbors' control input $u_j(t)$ at the time t. In this case, we replace $u_j(t)$ with $u_j(t-\delta)$ in (7), where δ is the time delay in the calculation.

We then consider the following feedforward and feedback gains:

$$K_{\mathrm{ff},i} = 1, \tag{8}$$

$$K_{\mathrm{fb},i} = \frac{1}{r_i} B^T P_i, \tag{9}$$

where $r_i > 0$ is a constant, $P_i \in \mathbb{R}^{N\times N}$ is the unique non-negative definite solution to the following algebraic Riccatti equation

$$P_i A + A^T P_i + \frac{1}{r_i} P_i B B^T P_i + Q_i = 0, \tag{10}$$

where $Q_i = D_i^T D_i \in \mathbb{R}^{N\times N}$ is a positive definite matrix, and the pair (A, D_i) is detectable.

Theorem 1: Suppose that $\bar{\mathcal{G}}$ contains a directed spanning tree rooted at the leading vehicle and $\mathcal{G}$ is a DAG. The controller (7) with the feedforward gain (8) and the feedback gain (9) guarantees the asymptotic stability of the system (1). In addition, the feedback gain (9) is optimal with respect to the following performance index:

$$\min_{K_{\mathrm{fb},i}} J_i = \frac{1}{2}\int_{t_0}^{+\infty} \left(\hat{x}_i(t)^T Q_i \hat{x}_i(t) + r_i \hat{u}_i(t)^2\right) \mathrm{d}t, \tag{11}$$

where

$$\hat{x}_i = \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}\left(\tilde{x}_i - \tilde{x}_j\right), \tag{12}$$

$$\hat{u}_i = \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}\left(u_i - u_j\right). \tag{13}$$

Proof: Combine (1) with (5), then we have:

$$\dot{\tilde{x}}_i = A\tilde{x}_i + B(u_i - u_0). \tag{14}$$

Combine (12) and (13) with (14), then we have:

$$\dot{\hat{x}}_i = A\hat{x}_i + B\hat{u}_i. \tag{15}$$

For the system (15) and the performance index (11), since $Q_i = Q_i^T > 0$ and $r_i > 0$, according to the LQR theory [29], the optimal linear feedback controller is:

$$\hat{u}_i = -\frac{1}{r_i} B^T P_i \hat{x}_i. \tag{16}$$

According to (13), we have:

$$\hat{u}_i = u_i - \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i} u_j. \tag{17}$$

Substitute (12) and (16) into (17), then we have:

$$u_i = \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i} u_j - \frac{1}{r_i} B^T P_i \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}\left(\tilde{x}_i - \tilde{x}_j\right). \tag{18}$$

By comparing (18) with (7), the optimality is proved.

Since the pair (A, B) is controllable and the pair (A, D_i) is detectable, we know that the system (15) is asymtotically stable, which means $\lim_{t\to+\infty}\hat{x}_i(t) = 0, \forall i = 1,2,\ldots,N$. Denote the topological ordering by $\{0, s_1, s_2, \ldots, s_N\}$, where $s_1, s_2, \ldots, s_N$ is a permutation of $1,2,\ldots,N$. Then we use the mathematical induction to prove the asymptotic stability of the system (1).

(a) For the vehicle s_1, since $\mathbb{I}_{s_1} = \{0\}$, we have $\lim_{t\to+\infty}\hat{x}_{s_1}(t) = \lim_{t\to+\infty}\tilde{x}_{s_1}(t) = 0$.

(b) Suppose that $\lim_{t\to+\infty}\tilde{x}_{s_k}(t) = 0$, then for the vehicle s_{k+1}, since $\mathbb{I}_{s_{k+1}} \subseteq \{0, s_1, s_2, \ldots, s_k\}$, we also have $\lim_{t\to+\infty}\hat{x}_{s_{k+1}}(t) = \lim_{t\to+\infty}\tilde{x}_{s_{k+1}}(t) = 0$.

This proves the asymptotic stability.

■

In (9) and (10), the parameters Q_i and r_i may be heterogeneous for each following vehicle. When it comes to homogeneous parameters, *i.e.*, $Q_i = Q, r_i = r, \forall i = 1,2,..,N$, we further have the second theorem of this paper.

Theorem 2: Suppose that $\bar{\mathcal{G}}$ contains a directed spanning tree rooted at the leading vehicle and $\mathcal{G}$ is a DAG. If $Q_i = Q, r_i = r, \forall i = 1,2,..,N$, which implies that $K_{\mathrm{fb},i} = K_{\mathrm{fb}}$, then the controller (7) with the feedforward gain (8) and the feedback gain (9) is equivalent to

$$u_i = u_0 - K_{\mathrm{fb}}\tilde{x}_i. \tag{19}$$

In addition, if $u_0 = 0$, then K_{fb} is optimal with respect to the following performance index:

$$\min_{K_{\mathrm{fb}}} J_i = \frac{1}{2}\int_{t_0}^{+\infty} \left(\tilde{x}_i(t)^T Q_i \tilde{x}_i(t) + r_i u_i(t)^2\right) \mathrm{d}t. \tag{20}$$

Proof: According to the LQR theory [29], the optimality is obvious. Denote the topological ordering by $\{0, s_1, s_2, \ldots, s_N\}$, where $s_1, s_2, \ldots, s_N$ is a permutation of $1,2,\ldots,N$. We only prove the equivalence of (7) and (19) using the mathematical induction.

(a) For the vehicle s_1, since $\mathbb{I}_{s_1} = \{0\}$, we have $u_{s_1} = u_0 - K_{\mathrm{fb}}\tilde{x}_{s_1}$.

(b) Suppose that $u_{s_k} = u_0 - K_{\mathrm{fb}}\tilde{x}_{s_k}$, then for the vehicle s_{k+1}, since $\mathbb{I}_{s_{k+1}} \subseteq \{0, s_1, s_2, \ldots, s_k\}$, we have

$$u_{s_{k+1}} = \frac{1}{\left|\mathbb{I}_{s_{k+1}}\right|}\sum_{j\in\mathbb{I}_{s_{k+1}}}\left(u_j - K_{\mathrm{fb}}\left(\tilde{x}_i - \tilde{x}_j\right)\right)$$

$$= \frac{1}{\left|\mathbb{I}_{s_{k+1}}\right|}\left[b_{s_{k+1}}\left(u_0 - K_{\mathrm{fb}}\tilde{x}_{s_{k+1}}\right) + \sum_{j=1}^{N} a_{s_{k+1},j}\left(u_j - K_{\mathrm{fb}}\left(\tilde{x}_i - \tilde{x}_j\right)\right)\right]$$

$$= \frac{1}{\left|\mathbb{I}_{s_{k+1}}\right|}\left[b_{s_{k+1}}\left(u_0 - K_{\mathrm{fb}}\tilde{x}_{s_{k+1}}\right) + \sum_{j=1}^{N} a_{s_{k+1},j}\left(u_0 - K_{\mathrm{fb}}\tilde{x}_{s_{k+1}}\right)\right]$$

$$= u_0 - K_{\mathrm{fb}}\tilde{x}_{s_{k+1}}.$$

This proves the equivalence.

■

Remark 3: According to (19), with homogeneous parameters, each following vehicle implicitly uses the leading vehicle's control input for feedforward, and the relative tracking error with respect to the leading vehicle for feedback, even though not all of the following vehicles can acquire the information of the leader. This property is not affected by the concrete communication topologies used in the platoons.

In the performance index (11), the average neighbor state error $\hat{x}_i$ and the average neighbor control input error $\hat{u}_i$ are penalized, which is not common in control design. Compared with (11), the performance index (20) is more practical in control design, considering that platoons are often operated at a preset constant velocity, which implies that $u_0 = 0$.

4. NUMERICAL SIMULATION

In this section, we present simulation results to demonstrate the effectiveness of the proposed control method. Consider a platoon consisting of one leading vehicle and seven following vehicles using four types of communication topologies, *i.e.*, PF, PLF, TPF, and TPLF, which are shown in Fig. 1. Note that for all these topologies, it holds that $\bar{\mathcal{G}}$ contains a directed spanning tree rooted at the leading vehicle and $\mathcal{G}$ is a DAG.

4.1 Simulation Setup

The initial states are $p_i(0) = -i \times d_0 + d_r$, $v_i(0) = v_0 + v_r$, $a_i(0) = 0$, where d_r and v_r are initial errors following the standard normal distribution, *i.e.*, $d_r \sim N(0, 1^2), v_r \sim N(0, 1^2)$. The control input (or the desired acceleration) of the leading vehicle is:

$$u_0(t) = \begin{cases} 0, & 0s \le t < 3s \\ 1, & 3s \le t < 15s \\ 0, & t \ge 15s \end{cases}, (m/s^2). \tag{21}$$

The other simulation parameters are listed in Table 1.

We compare the proposed feedforward-feedback controller (denoted by FFFB) with the feedback controller (denoted by FB) similar to the one designed in [27]:

$$u_i = -K_{\text{fb},i} \frac{1}{|\mathbb{I}_i|} \sum_{j \in \mathbb{I}_i} (\tilde{x}_i - \tilde{x}_j), \tag{22}$$

which contains no feedforward term but shares the same feedback gain $K_{\text{fb},i}$. For the FFFB controller, we also consider the time delay in the calculation by replacing $u_j(k)$ with $u_j(k-1)$ at the time k (see ***Remark 2***) in the discrete-time simulations. This delayed FFFB controller is denoted by dFFFB for convenience. The quantitative performance of these three controllers is measured with the practical performance index (20).

4.2 Simulation Results

The profiles of spacing error are shown in Fig. 3. It is clear that when the leading vehicle moves at a constant velocity, the platoons with three types of controllers are all asymptotically stable. However, when the leading vehicle accelerates with a constant non-zero control input, both the FFFB and dFFFB controllers can still guarantee the asymptotically tracking, while the FB controller cannot. This demonstrates the advantage of the feedforward design.

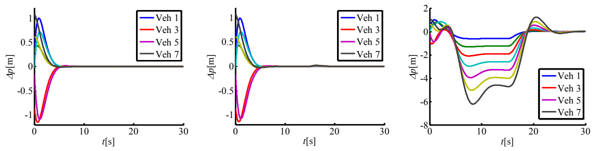
The sums of performance indices of the seven following vehicles are listed in Table 2. It is obvious that the FFFB and dFFFB controllers outperform the FB controller for all the four types of communication topologies. It is also observed that the differences in the performance of the FFFB and dFFFB controllers are neglectable.

Table 1 Simulation parameters

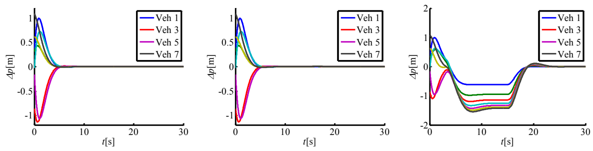
parameter	value
N	8
τ	0.3
d_0	20 (m)
v_0	10 (m/s)
Q_i	$\text{diag}\{3,2,1\} + 0.2 \times i \times I_3$
r_i	$1 + 0.2 \times i$

Table 2 Performance index

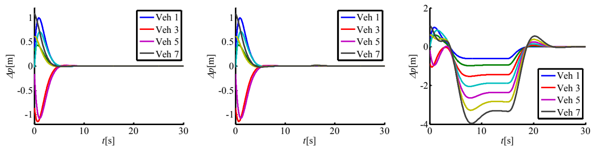
communication topology	controller		
	FFFB	dFFFB	FB
PF	120.01	123.81	1899.28
PLF	120.09	120.49	363.39
TPF	120.02	120.86	974.64
TPLF	120.06	120.41	456.40



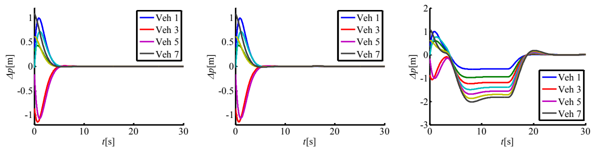
(a) PF



(b) PLF



(c) TPF



(d) TPLF

Fig. 3 Profiles of spacing error
(left: FFFB; middle: dFFFB; right: FB)

5. CONCLUSIONS

This paper has introduced a behavioral cooperation method for multiple connected vehicle systems with directed acyclic interactions. A novel feedforward-feedback control scheme has been designed for consensus-based platoon control. By using the LQR theory, it is proved that the proposed feedforward-feedback controller guarantees the asymptotic stability of the system and is optimal with respect to a specific quadratic performance index, since it implicitly uses the leader's information. The advantage of the proposed control method is validated through numerical simulations.

Future work includes the study of other types of feedforward strategies. Besides, the time delay in communication and the string stability also deserve further consideration.

ACKNOWLEDGEMENT

This research is supported by NSF China with 51575293, 51622504, 51475254, 51605245 and 51625503, National Key R&D Program of China with 2016YFB0100906, National Sci&Tech Major Project of China with 2016ZX03002019, and International Sci&Tech Cooperation Program of China under 16YFE0102200.

REFERENCES

[1] Shladover, S. E. "PATH at 20 - History and major milestones," *IEEE Trans. Intell. Transp. Syst.*, Vol. 8, No. 4, 2007, pp. 584–592.

[2] Coelingh, E., and Solyom, S. "All Aboard the Robotic Road Train," *IEEE Spectr.*, Vol. 49, No. 11, 2012, pp. 34–39.

[3] Kianfar, R., Augusto, B., Ebadighajari, A., et al. "Design and Experimental Validation of a Cooperative Driving System in the Grand Cooperative Driving Challenge," *IEEE Trans. Intell. Transp. Syst.*, Vol. 13, No. 3, 2012, pp. 994–1007.

[4] Tsugawa, S., Jeschke, S., and Shladover, S. E. "A Review of Truck Platooning Projects for Energy Savings," *IEEE Trans. Intell. Transp. Syst.*, Vol. 1, No. 1, 2016, pp. 68–77.

[5] Li, S. E., Zheng, Y., Li, K., et al. "Dynamical Modeling and Distributed Control of Connected and Automated Vehicles: Challenges and Opportunities," *IEEE Intell. Transp. Syst. Mag.*, 2017, Vol. 9, No. 3, pp. 46–58.

[6] Ploeg, J., van de Wouw, N., and Nijmeijer, H. "L-p String Stability of Cascaded Systems: Application to Vehicle Platooning," *IEEE Trans. Control Syst. Technol.*, Vol. 22, No. 2, 2014, pp. 786–793.

[7] Liang, K.-Y., Martensson, J., and Johansson, K. H. "Heavy-Duty Vehicle Platoon Formation for Fuel Efficiency," *IEEE Trans. Intell. Transp. Syst.*, Vol. 17, No. 4, 2016, pp. 1051–1061.

[8] Kwon, J. W., and Chwa, D. "Adaptive Bidirectional Platoon Control using a Coupled Sliding Mode Control Method," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, No. 5, 2014, 2040–2048.

[9] Guo, X., Wang, J., Liao, F., et al. "Distributed Adaptive Integrated-sliding-mode Controller Synthesis for String Stability of Vehicle Platoons," *IEEE Trans. Intell. Transp. Syst.*, Vol. 17, No. 9, 2016, 2419–2429.

[10] Dunbar, W. B., and Caveney, D. S. "Distributed Receding Horizon Control of Vehicle Platoons: Stability and String Stability," *IEEE Trans. Autom. Control*, Vol. 57, No. 3, 2012, pp. 620–633.

[11] Zheng, Y., Li, S. E., Li, K., et al. "Distributed Model Predictive Control for Heterogeneous Vehicle Platoons Under Unidirectional Topologies," *IEEE Trans. Control Syst. Technol.*, Vol. 25, No. 3, 2017, pp. 899–910.

[12] Li, S. E., Gao, F., Li, K., et al. "Robust Longitudinal Control of Multi-Vehicle Systems--A Distributed H-Infinity Method," *IEEE Trans. Intell. Transp. Syst.*, 2018 (early access).

[13] Ploeg, J., Shukla, D. P., van de Wouw, N., et al. "Controller Synthesis for String Stability of Vehicle Platoons," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, No. 2, 2014, pp. 854–865.

[14] Zheng, Y., Li, S. E., Li, K., and Ren, W. "Platooning of connected vehicles with undirected topologies: Robustness analysis and distributed h-infinity controller synthesis," *IEEE Trans. Intell. Transp. Syst.*, Vol. 19, No. 5, pp. 1353–1364, 2018.

[15] Li, S. E., Li, R., Wang, J., et al. "Stabilizing Periodic Control of Automated Vehicle Platoon with Minimized Fuel Consumption," *IEEE Trans. Transport. Electrific.*, Vol. 3, No. 1, 2017, pp. 259–271.

[16] Li, S. E., Deng, K., Zheng Y., and Peng, H. "Effect of Pulse-and-Glide Strategy on Traffic Flow for a Platoon of Mixed Automated and Manually Driven Vehicles," *Computer-Aided Civil and Infrastructure Eng.*, Vol. 30, No. 11, 2015, pp. 892–905.

[17] Zhou, J., and Peng, H. "Range Policy of Adaptive Cruise Control Vehicles for Improved Flow Stability and String Stability," *IEEE Trans. Intell. Transp. Syst.*, Vol. 6, No. 2, 2005, pp. 229–237.

[18] Milanes, V., Shladover, S. E., Spring, J., et al. "Cooperative Adaptive Cruise Control in Real Traffic Situations," *IEEE Trans. Intell. Transp. Syst.*, Vol. 15, No. 1, 2014, pp. 296–305.

[19] Ge, J. I., and Orosz, G. "Optimal Control of Connected Vehicle Systems with Communication Delay and Driver Reaction Time," *IEEE Trans. Intell. Transp. Syst.*, Vol. 18, No. 8, 2017, pp. 2056–2070.

[20] Zhang, L., and Orosz, G. "Motif-based Design for Connected Vehicle Systems in Presence of Heterogeneous Connectivity Structures and Time Delays," *IEEE Trans. Intell. Transp. Syst.*, Vol. 17, No. 6, 2016, 1638–1651.

[21] Ploeg, J., Semsar-Kazerooni, E., Lijster, G., et al. "Graceful Degradation of Cooperative Adaptive Cruise Control," *IEEE Trans. Intell. Transp. Syst.*, Vol. 16, No. 1, 2015, pp. 488–497.

[22] Naus, G. J. L., Vugts, R. P. A., Ploeg, J., et al. "String-Stable CACC Design and Experimental Validation: A Frequency-Domain Approach," *IEEE Trans Veh Technol*, Vol. 59, No. 9, 2010, pp. 4268–4279.

[23] Al-Jhayyish, A. M. H., and Schmidt, K. W. "Feedforward Strategies for Cooperative Adaptive Cruise Control in Heterogeneous Vehicle Strings," *IEEE Trans. Intell. Transp. Syst.*, Vol. 19, No. 1, 2018, pp. 113–122.

[24] Ren, W., and Beard, R. W. "Distributed Consensus in Multi-vehicle Cooperative Control," 2008, London: Springer.

[25] Fax, J. A., and Murray, R. M. "Information Flow and Cooperative Control of Vehicle Formations," *IEEE Trans. Autom. Control*, Vol. 49, No. 9, 2004, pp. 1465–1476.

[26] Li, Z., Duan, Z., Chen, G., et al. "Consensus of Multiagent Systems and Synchronization of Complex Networks: A Unified Viewpoint," *IEEE Trans. Circuits Syst. I, Reg. Papers*, Vol. 57, No. 1, 2010, pp. 213–224.

[27] Zheng, Y., Li, S. E., Wang, J., et al. "Stability and Scalability of Homogeneous Vehicular Platoon: Study on the Influence of Information Flow Topologies," *IEEE Trans. Intell. Transp. Syst.*, Vol. 17, No. 1, 2016, pp. 14–26.

[28] Zheng, Y., Bian, Y., Li, S., Li, S. E. "Cooperative Control of Heterogeneous Connected Vehicles with Directed Acyclic Interactions," *arXiv preprint*, arXiv:1805.04304, 2018.

[29] Anderson, B. D. and Moore, J. B. "Optimal Control: Linear Quadratic Methods," 2007, Courier Corporation.