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Reducing Time Headway for Platoons of Connected Vehicles via Multiple-Predecessor Following

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***Abstract*— In a platoon of connected vehicles, a lower time headway can reduce the inter-vehicle distance, thus leading to higher traffic capacity. This paper discusses an approach to reduce time headway for string stable platoons via multiple-predecessor following (MPF). First, the platoon system is formulated using a third-order linear model and a linear feedback controller under the MPF topology. Then, we introduce a new definition of desired inter-vehicle distances using the constant time headway (CTH) policy, which avoids inconsistency in inter-vehicle distances. Under the proposed definition, we present a sufficient condition to guarantee string stability by analyzing the feasible region of feedback gains. It is proved that there exist string stable feedback gains if the time headway is lower bounded. The result indicates that increasing the number of predecessors can reduce the time headway for string stability, which in turn increases the road capacity. Numerical simulations validate the theoretical results.**

***Index Terms*— vehicular platoon, constant time headway, multiple-predecessor following, string stability**

I. INTRODUCTION

The increase of car ownership poses a high demand on road throughput. One promising approach to increase highway traffic efficiency is the coordination of multiple connected vehicles using onboard sensors and V2V (vehicle-to-vehicle) communications, yielding the so-called vehicular platoons [1]. This technique can further improve fuel efficiency by reducing inter-vehicle distance [2] and achieving cooperation in the servo loop [3]. A review for the recent advances in platoon control can be found in [4], [5].

In a platoon, multiple connected vehicles are coordinated to drive in a one-dimensional formation. One important property is the notion of *string stability*, *i.e.*, the attenuation of the effects of disturbances along the platoon [6]. There are different types of definitions for string stability, such as the $\mathcal{L}_2$ [7], [8], $\mathcal{L}_p$ [9], $\mathcal{L}_\infty$ [10] and head-to-tail [11] string stability. It is well-known that string stability is highly related to the range policy in a platoon, *i.e.*, how the desired inter-vehicle distance are defined. In the literature, commonly used range policies include: 1) constant spacing (CS) policy, and 2) constant time headway (CTH) policy. For the CS policy, the desired inter-vehicle distance is a constant value, which can reduce the platoon length and thus improve transport throughput. However, for the CS policy, string stability cannot be achieved using an identical linear controller for the predecessor-following (PF) topology [6], [12] and the bi-directional (BD) topology [13]. One solution for string stability is to introduce the leader's information for every following vehicle, yielding the predecessor-leader following (PLF) topology [12]. However, the PLF topology requires communication between the leader and every followers, which is burdensome when the platoon length becomes large. Another method to mitigate string instability is to use asymmetric controller [14], which instead requires that the feedback gains increase with the platoon size. A more recent discussion on the CS policy can be found in [15]. On the other hand, for the CTH policy, the desired inter-vehicle distance has a linear relationship with the host vehicle's velocity, which agrees with human drivers' characteristics. In this case, string stability can be achieved without relying on the leader's information [7], but the transport throughput may be impacted since the inter-vehicle distance increases as the velocity grows. We note that nonlinear range policies may also be applied in platoons, *e.g.*, quadratic policy [16], adaptive policy [17] and delay-based policy [18].

In recent years, the rapid development of V2V techniques enriches information flow topologies for platoon design [5]. Various information topologies bring benefits as well as challenges to the analysis and design of multivehicle systems, thus attracting increasing research attention. For example, a separation principle was introduced to use the Laplacian matrix to determine formation stability in [19]. A four-component framework has been proposed to study the influence of information topologies on the stability [20], scalability and robustness [21] of vehicle platoons. Similar methods are also used in [22], [23] based on the decomposition of the information matrix. These studies provide certain insights on the influence of V2V communication on platoons with the CS policy. However, string stability may still be unsatisfied for platoons with the CS policy [15]. Therefore, it remains an important topic to address the effect of V2V communication on platoons with the CTH policy.

In this paper, we investigate whether V2V communication can reduce the allowable time headway for a string stable platoon. Two recent studies have shown some positive answers [24], [25]. Inspired by these two studies, we aim to find the minimum allowable time headway to guarantee the

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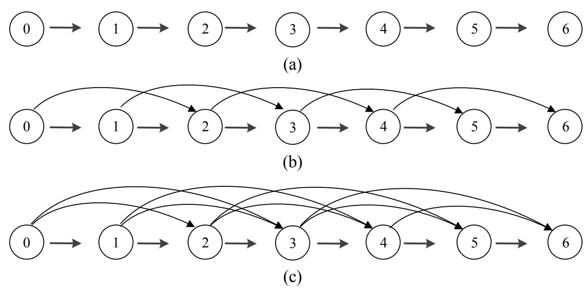


Fig. 1. Examples of the MPF topology (r denotes the number of predecessors). (a) $r = 1$; (b) $r = 2$; (c) $r = 3$.

string stability. Our main motivation is that different desired inter-vehicle distances may be defined for platoons with V2V communication, and the results in [24], [25] are suitable for a straightforward case only. In the CTH policy, time headway typically denotes the time taken for the front bumper of a vehicle to arrive at the position of its predecessor's front bumper. Therefore, the desired inter-vehicle distance directly depends on the host vehicle's velocity in a platoon with the PF topology. For general information flow topologies, such as the multiple-predecessor following (MPF) and the multiple-predecessor-leader following (MPLF), the definition of desired inter-vehicle distances may be different. In [26], the leading vehicle's velocity is used to define desired inter-vehicle distances for platoons with the MPLF topology. In [24], [25], the host vehicle's velocity is used to define desired inter-vehicle distances for platoons with the MPF topology. However, these definitions either rely on the leading vehicle's information or may cause inconsistency in inter-vehicle distances.

In this paper, we introduce a new definition of desired inter-vehicle distances using the CTH policy for general information flow topologies, which avoids inconsistency in desired inter-vehicle distances. Then, we present analytical results to quantify the influence of V2V communication on time headway for platoons. Specifically, we derive a sufficient condition to guarantee string stability, which gives a lower bound of allowable time headway for platoons with the MPF topology. It is shown that using the information of multiple predecessors can reduce allowable time headway for string stability. The reduced time headway can, in turn, increase the transport capacity and improve fuel efficiency. Our results extend that of [24], [25] in terms of the definition of the CTH policy.

The remainder of this paper is organized as follows. Section II presents the system modeling. Section III defines the time headway policy for platoons with the MPF topology. String stability is analyzed in Section IV, followed by numerical simulations in Section V. We conclude this paper in Section VI.

II. System Modeling

We consider a homogeneous platoon of connected vehicles that consists of 1 leader and N followers, indexed by 0 and $1, 2, \ldots, N$. The road is assumed to be straight and flat, so the lateral vehicle motion is neglected for convenience. The control objective is to coordinate the longitudinal motion of the vehicles so that they keep a desired inter-vehicle distance while maintaining a desired velocity. As suggested in [5], [20], we model a vehicular platoon from the following aspects: 1) vehicle dynamics, which describe the longitudinal behavior of each vehicle; 2) information flow topology, which defines how vehicles exchange information with each other; 3) formation geometry, which depicts the desired inter-vehicle distance; 4) distributed controller, which implements feedback control law on each vehicle based on local information.

A. Vehicle Dynamics

The dynamics of the leading and following vehicles are:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = a_i, \\ \tau \dot{a}_i + a_i = u_i, \end{cases} \qquad i \in \{0, 1, 2, \ldots, N\}, \tag{1}$$

where p_i, v_i, and a_i denote the position, velocity, and acceleration, respectively; $\tau > 0$ is the inertial time lag in the powertrain. In (1), it is assumed each vehicle is equipped with a low-level acceleration controller that regulate a_i according to u_i. The low-level controller is modeled as a first-order lag system with a time constant τ. Note that this model is simple yet accurate enough for the platoon level synthesis, which has been widely used in the literature, *e.g.*, [16], [20], [24].

B. Information Flow Topology

We consider the MPF topology where each vehicle is connected to multiple immediate predecessors. Here the number of connected predecessors is denoted by $r \in \mathbb{Z}^+$. In particular, for vehicle i, $1 \leq i < r$, it can acquire the information of vehicles $0, 1, 2, \ldots, i-1$. Fig. 1 gives several examples of the MPF topology.

C. Formation Geometry

We assume that the leading vehicle is running at a constant velocity, *i.e.*, $a_0(t) = 0$, which is also considered in [20]. Then the control objective is formulated as: $\forall i \in \{1, 2, \ldots, N\}$,

$$\begin{cases} \lim\limits_{t \to +\infty} \|p_i(t) - p_0(t) + d_{i,0}(t)\| = 0, \\ \lim\limits_{t \to +\infty} \|v_i(t) - v_0(t)\| = 0, \\ \lim\limits_{t \to +\infty} \|a_i(t) - a_0(t)\| = 0, \end{cases}$$

where $d_{i,0}(t) > 0$ is the desired inter-vehicle distance between vehicle 0 and vehicle i.

D. Distributed Controller

The linear feedback controller is designed as:

$$u_i = -\sum_{j=i-1}^{i-r} \big(k_p(\tilde{p}_i - \tilde{p}_j) + k_v(\tilde{v}_i - \tilde{v}_j) + k_a(\tilde{a}_i - \tilde{a}_j)\big), \tag{2}$$

where k_p, k_v, and k_a are feedback gains; $\tilde{p}_i$, $\tilde{v}_i$, and $\tilde{a}_i$, defined in (6), are the position, velocity, and acceleration tracking errors, respectively. Compared with [24], we apply the acceleration error with respect to predecessors for feedback, instead of using the acceleration of predecessors for feedforward only, which turns out to be equivalent after a proper transformation.

III. Definition of Constant Time Headway Policy

For the CTH policy in the PF topology, the desired inter-vehicle distance between vehicles i and $i-1$ is defined as

$$d_{i,i-1} = hv_i + D, \tag{3}$$

where h is the time headway and D is the standstill distance.

When it comes to general topologies, the definitions of desired inter-vehicle distance using the CTH policy are not consistent in the literature. For example, in [26], the desired distance between vehicle i and vehicle $i-l$ is defined as $d_{i,i-l} = (\sum_{k=i-l}^{i-1} h_k)v_0 + lD$, where v_0 is the velocity of the leading vehicle, h_k is the time headway of vehicle $k-1$ with respect to vehicle k. This definition is based on the leading vehicle's velocity, implying that each vehicle can obtain the leader's information. In [24], [25], the desired inter-vehicle distance between i and vehicle $i-l$ is defined as $d_{i,i-l} = lhv_i + lD$, where h is the time headway of vehicle i with respect to vehicle $i-l$. This definition is based on the velocity of the host vehicle. However, this will result in inconsistency in inter-vehicle distances for the transient process, *i.e.*, $d_{i,k} \neq d_{i,j} + d_{j,k}$ if $v_i \neq v_j$.

In this paper, we directly extend the definition of the CTH policy in the PF topology (3) by adding the desired inter-vehicle distances of all predecessors together, *i.e.*,

$$d_{i,i-l}(t) = \sum_{k=i}^{i-l+1} \big(hv_k(t) + d_k\big), \tag{4}$$

where $h \leq 0$ and $d_k > 0$ are the time headway and desired standstill gap of vehicle k with respect to vehicle $k-1$, respectively. The definition (4) relies on the velocities of $l-1$ immediate predecessors, which can avoid the inconsistency in desired inter-vehicle distances. It is not difficult to check that (similar to [17]):

$$d_{i,k}(t) = d_{i,j}(t) + d_{j,k}(t), \forall t \geq t_0$$

Based on the definition (4), the desired distance between vehicle i and vehicle 0 is

$$d_{i,0} = \sum_{k=1}^{i} (hv_k + d_k). \tag{5}$$

Then the tracking error is defined as:

$$\begin{cases} \tilde{p}_i = p_i + \sum\limits_{k=1}^{i} (hv_k + d_k) - p_0, \\ \tilde{v}_i = v_i - v_0, \\ \tilde{a}_i = a_i - a_0. \end{cases} \tag{6}$$

Remark 1: Note that although $\tilde{p}_i$, $\tilde{v}_i$, and $\tilde{a}_i$ in (6) are defined using the leading vehicle's information. However, the calculations of $\tilde{p}_i - \tilde{p}_j$, $\tilde{v}_i - \tilde{v}_j$, and $\tilde{a}_i - \tilde{a}_j$ in (2) only require the information of vehicles $i, i-1, \ldots, j$ $(j < i)$ or vehicles $i, i+1, \ldots, j$ $(j > i)$.

IV. String Stability Analysis

In this section, we first formulate the transfer function of distance errors and then present the string stability criterion.

A. Transfer Function of Distance Errors

Here the distance error is defined as:

$$e_i = p_i - p_{i-1} + hv_i + d_i.$$

According to (1), we know:

$$u_i = \tau \dot{a}_i + a_i = \tau \dddot{p}_i + \ddot{p}_i. \tag{7}$$

By substituting (7) into (2), we have:

$$\tau \dddot{p}_i + \ddot{p}_i = -\sum_{l=1}^{r} \Big(k_p\big(p_i - p_{i-l} + d_{i,i-l}\big) + k_v(v_i - v_{i-l}) + k_a(a_i - a_{i-l})\Big), \tag{8}$$

and

$$\tau \dddot{p}_{i-1} + \ddot{p}_{i-1} = -\sum_{l=1}^{r} \Big(k_p\big(p_{i-1} - p_{i-1-l} + d_{i-1,i-1-l}\big) + k_v(v_{i-1} - v_{i-l-1}) + k_a(a_{i-1} - a_{i-l-1})\Big), \tag{9}$$

where $d_{i,j} = \sum_{k=j+1}^{i} (hv_k + d_k)$. In addition, the derivative of (8) is

$$\tau \dddot{v}_i + \ddot{v}_i = -\sum_{l=1}^{r} \Big(k_p\big(v_i - v_{i-l} + \sum_{i}^{k=i-l+1} (ha_k)\big) + k_v(a_i - a_{i-l}) + k_a(\dot{a}_i - \dot{a}_{i-l})\Big). \tag{10}$$

Then by calculating (8) + $h \times$ (10) − (9), we have:

$$\tau \dddot{e}_i + (rk_a + 1)\ddot{e}_i + r(k_v + k_p h)\dot{e}_i + rk_p e_i = \sum_{l=1}^{r} \Big(k_p e_{i-l} + \big(k_v - k_p h(r-l)\big)\dot{e}_{i-l} + k_a \ddot{e}_{i-l}\Big). \tag{11}$$

We assume that the initial error is zero, then the Laplace transform of (11) becomes the following form:

$$E_i(s) = \sum_{l=1}^{r} H_l(s) E_{i-l}(s),$$

where $E_k(s)$ is the Laplace transform of $e_k(t)$, and

$$H_l(s) = \frac{k_a s^2 + \big(k_v - k_p h(r-l)\big)s + k_p}{\tau s^3 + (rk_a + 1)s^2 + r(k_v + k_p h)s + rk_p}. \tag{12}$$

For $H_l(s)$, it is required that the denominator should be Hurwitz, which implies closed-loop stability. Then we have the following proposition.

Proposition 1: For a platoon of vehicles with dynamics given in (1), formation geometry given in (5), and distributed controller given in (2), we assume that the number of predecessors is r in the MPF topology. Then the platoon is asymptotically stable if and only if:

$$\begin{cases} r > 0, \\ k_a > -\frac{1}{r}, \\ k_p > 0, \\ k_v > (\frac{\tau}{1+k_a r} - h)k_p. \end{cases} \quad (13)$$

The proof is straightforward by analyzing the stability of the denominator of (12) using the Routh-Hurwitz criterion. We omit it for brevity here. Note that the result in Proposition 1 is a direct extension of the stability region in [20] that deals with platoons with the CS policy.

Remark 2: According to Proposition 1, the lower bound of time headway for closed-loop stability is

$$h > h_{\min,1} := \frac{\tau}{1+k_a r} - \frac{k_v}{k_p}.$$

For simplicity, we denote two sets of (k_p, k_v) as

$$B_1 := \{(k_p, k_v) | k_p > 0\},$$
$$B_2 := \left\{(k_p, k_v) \middle| k_v - \left(\frac{\tau}{1+k_a r} - h\right) k_p > 0\right\}.$$

B. String Stability Criterion

For $\mathcal{L}_2$ string stability, a sufficient condition [24] is:

$$\sum_{l=1}^{r} \|H_l(j\omega)\|_\infty \leq 1. \quad (14)$$

It is not difficult to check that (14) implies the attenuation of distance errors in the sense of $\|e_i\|_2^2 \leq \frac{1}{r}\sum_{l=1}^{r}\|e_{i-l}\|_2^2$. Note that when $r = 1$, (14) becomes a necessary and sufficient condition for $\mathcal{L}_2$ string stability [9]. Then, we are ready to present the main theorem on the string stability criterion.

Theorem 1: For a platoon of connected vehicles with dynamics given in (1), formation geometry given in (5), and distributed controller given in (2), we assume that in the MPF topology the number of predecessors is $r \geq 1$. Then there exists a set of feedback gains $[k_p, k_v, k_a]$ such that (13) and (14) hold if:

$$h \geq h_{\min,2} := \frac{2\tau}{2k_a r + 1}, k_a > -\frac{1}{2r}. \quad (15)$$

Proof: According to (12), we have:

$$|H_l(j\omega)|^2 = \frac{(k_p - k_a\omega^2)^2 + \left(k_v - k_p h(r-l)\right)^2\omega^2}{\left(r(k_v + k_p h)\omega - \tau\omega^3\right)^2 + \left(rk_p - (rk_a+1)\omega^2\right)^2}.$$

Since $\lim_{\omega\to 0^+} |H_l(j\omega)| = \frac{1}{r}$, we know that (14) holds if and only if:

$$\max_{l, 1\leq l\leq r} \|H_l(j\omega)\|_\infty^2 = \max_{1\leq l\leq r} \sup_{\omega>0} |H_l(j\omega)|^2 \leq \frac{1}{r^2}.$$

Additionally, since $|H_l(j\omega)|^2$ is a quadratic function of l, we have:

$$\max_{l,1\leq l\leq r} \sup_{\omega>0} |H_l(j\omega)|^2 = \max\left\{\sup_{\omega>0}|H_1(j\omega)|^2, \sup_{\omega>0}|H_r(j\omega)|^2\right\}.$$

Therefore, we know that (14) holds if and only if:

$$\begin{cases} \sup_{\omega>0} |H_1(j\omega)|^2 \leq \frac{1}{r^2}, \\ \sup_{\omega>0} |H_r(j\omega)|^2 \leq \frac{1}{r^2}. \end{cases}$$

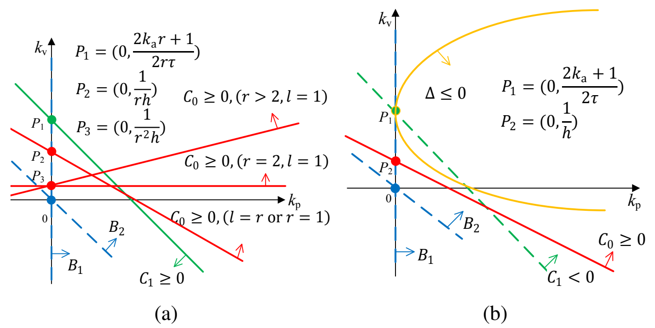


Fig. 2. The feasible region of (k_p, k_v).

Upon defining:

$$N_l = (k_p - k_a\omega^2)^2 + \left(k_v - k_p h(r-l)\right)^2\omega^2,$$
$$D_l = \left(r(k_v + k_p h)\omega - \tau\omega^3\right)^2 + \left(rk_p - (rk_a+1)\omega^2\right)^2,$$

we know that $|H_l(j\omega)|^2 \leq 1/r^2$ is equivalent to:

$$D_l - r^2 N_l = \omega^2\left(C_2\omega^4 + C_1\omega^2 + C_0\right) \geq 0, \quad (16)$$

where

$$C_2 = \tau^2, \quad (17)$$
$$C_1 = 2k_a r + 1 - 2r\tau(k_v + hk_p), \quad (18)$$
$$C_0 = k_p r\left(k_p r\left(1 - (l-r)^2\right)h^2 + 2k_v r(1+r-l)h - 2\right). \quad (19)$$

Then we only need to consider the bi-quadratic function $C_2\omega^4 + C_1\omega^2 + C_0$. We denote the discriminant by:

$$\Delta = C_1^2 - 4C_2C_0.$$

Since $C_2 > 0$, upon defining the following two conditions:

$$C_0 \geq 0, \quad C_1 \geq 0, \quad (20)$$
$$C_0 \geq 0, \quad C_1 < 0, \quad \Delta \leq 0, \quad (21)$$

we have:

$$(20) \text{ or } (21) \text{ holds} \Longleftrightarrow (16) \text{ holds}. \quad (22)$$

Besides, from (19), we know $h > 0$ is necessary for $C_0 \leq 0$.

We consider the sufficient conditions (20). By combining (17)-(19) with (13), we have:

$$C_0 \geq 0 \Longleftrightarrow k_v \geq \begin{cases} -\frac{h}{2}k_p + \frac{1}{rh}, & l = r, \\ \frac{(r-2)h}{2}k_p + \frac{1}{r^2 h}, & l = 1, \end{cases} \quad (23)$$

$$C_1 \geq 0 \Longleftrightarrow k_v \leq -hk_p + \frac{2k_a r + 1}{2r\tau}. \quad (24)$$

Given a set of parameters h, k_a, r, and τ, the feasible region of (k_p, k_v) given by (13) and (23)-(24) is shown in Fig. 2(a). Then we know that there exist (k_p, k_v) such that (20) holds if and only if the k_v coordinate of P_1 is greater than that of P_2, *i.e.*,

$$\frac{2k_a r + 1}{2r\tau} > \frac{1}{rh},$$

which is equivalent to (15). This completes the proof. ■

Remark 3: According to Theorem 1, k_a can be negative, which means positive acceleration error feedback. Besides, increasing k_a or r can reduce the lower bound of h, which indicates a lower inter-vehicle distance and higher traffic capacity.

In Theorem 1, a sufficient condition is derived for (14). In particular, when $r = 1$, we can further get a necessary and sufficient condition.

Theorem 2: For a platoon of connected vehicles with dynamics given in (1), formation geometry given in (5), and distributed controller given in (2), we assume that in the MPF topology the number of predecessors is $r = 1$. Then there exists a set of feedback gains $[k_p, k_v, k_a]$ such that the platoon is string stable if and only if:

$$h \geq h_{\min,2} = \frac{2\tau}{2k_a + 1}, k_a > -\frac{1}{2}. \tag{25}$$

Proof: We consider the sufficient conditions (21). By combining (17)-(19) with (13), we have:

$$C_0 \geq 0 \Longleftrightarrow k_v \geq -\frac{h}{2}k_p + \frac{1}{h}, \tag{26}$$

$$C_1 < 0 \Longleftrightarrow k_v > -hk_p + \frac{2k_a + 1}{2\tau}, \tag{27}$$

$$\Delta \leq 0 \Longleftrightarrow \big(2\tau - h(2k_a + 1)\big)k_p \leq -\tau\Big(k_v - \frac{2k_a + 1}{2\tau}\Big)^2. \tag{28}$$

According to (13), we know that $k_p > 0$. Then according to (28), we know that $h \geq \frac{2\tau}{2k_a+1}$, $2k_a + 1 > 0$.

1) When $h = \frac{2\tau}{2k_a+1}$, according to (28), we have $k_v = \frac{2k_a+1}{2\tau}$. It is easy to check that there exist (k_p, k_v) such that (21) holds.

2) When $h > \frac{2\tau}{2k_a+1}$, given a set of parameters h, k_a, and τ, the feasible region of (k_p, k_v) given by (13) and (26)-(28) is shown in Fig. 2(b). Then we know there exist (k_p, k_v) such that (21) holds.

Since (25) implies (15) when $r = 1$, by taking into account (22), we know (25) is a necessary and sufficient condition for (13) and (14). In addition, since $r = 1$, (14) becomes a necessary and sufficient condition for string stability. This completes the proof. ■

Remark 4: In [24], [27], similar results are also derived. In particular, when $r = 1$, Theorem 2 agrees with the result in [24]; if the acceleration is not available, *i.e.*, $k_a = 0$, Theorem 2 is consistent with the result in [27], which indicates that the employable time headway is lower bounded by 2τ for string stability. However, when $r > 1$, since the definition of the CTH policy in this study is different from that in [24], the results of Theorem 1 has a different meaning compared to [24].

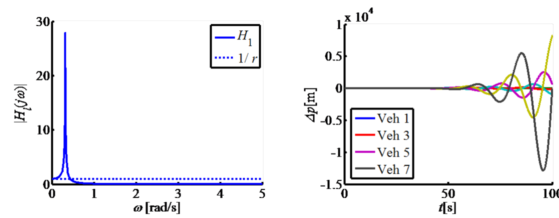
In addition, we have the following corollary, which is consistent with the results in [6], [12], [15].

Corollary 1: For a platoon of vehicles with dynamics given in (1), formation geometry given in (5), and distributed controller given in (2), we assume that the number of predecessors is r in the MPF topology. If the formation geometry is a CS policy ($h = 0$), then (14) will never hold.

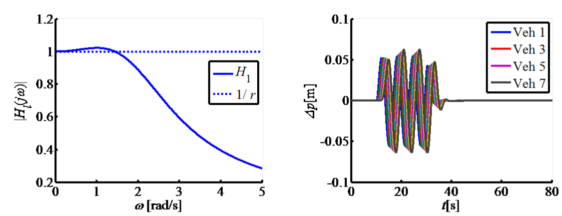
TABLE I
SIMULATION PARAMETERS

Platoon parameters			
N	d_i[m]	τ[s]	v_0[m/s]
7	10	0.5	10

Control parameters								
Fig.	r	k_p	k_v	k_a	ω[rad/s]	$h_{\min,1}$[s]	$h_{\min,2}$[s]	h[s]
3(a)	1	0.1	0.01	0.01	1.0	0.395	0.980	0.316
3(b)	1	0.1	2.51	0.51	1.0	-24.7	0.495	0.396
3(c)	1	0.1	1.65	0.51	1.0	-16.2	0.495	0.594
4(a)	3	0.1	0.01	0.68	1.6	0.065	0.198	0.052
4(b)	3	0.1	2.52	0.84	1.6	-25.0	0.165	0.132
4(c)	3	0.1	1.67	0.84	1.6	-16.6	0.165	0.198



(a)



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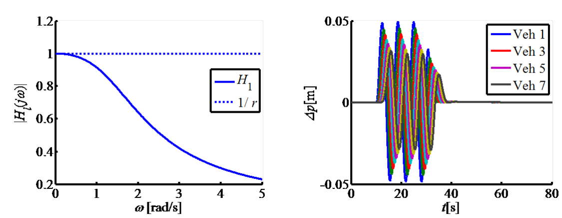


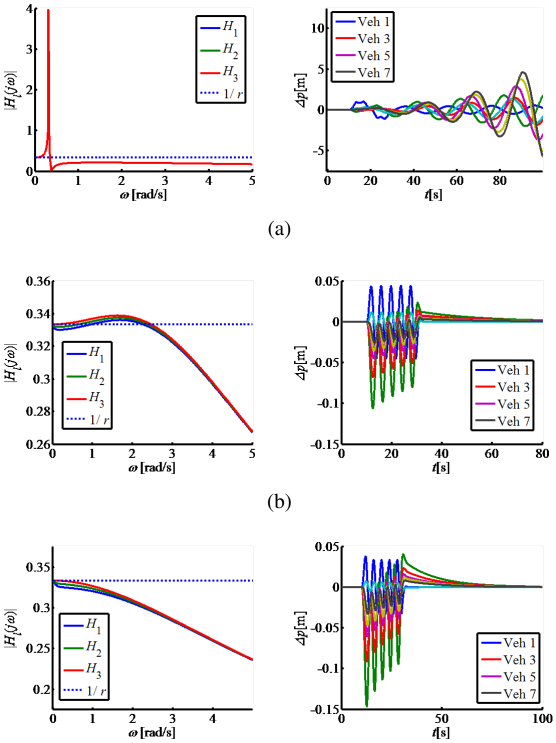
(c)

Fig. 3. Simulation results for $r = 1$.

V. NUMERICAL SIMULATIONS

This section presents numerical simulations to validate the proposed theorems. We consider platoon control in three cases, *i.e.*, 1) $h \leq h_{\min,1}$, 2) $h_{\min,1} < h \leq h_{\min,2}$, and 3) $h > h_{\min,2}$, and the number of predecessors is $r = 1$ or $r = 3$. The initial errors are assumed to be zero, and sinusoidal disturbances with frequency ω are imposed on the control input of the leading vehicle during 10s - 30s. The simulation parameters are listed in TABLE I. Simulation results are shown in Fig. 3 and Fig. 4.

As shown in Fig. 3(a) and Fig. 4(a), when $h \leq h_{\min,1}$, there are peaks in the magnitude-frequency diagrams, which correspond to the poles of $H_l(j\omega)$. In this case, the platoons are not asymptotically stable, which confirms Proposition 1. As shown in Fig. 3(b) and Fig. 4(b), when $h_{\min,1} < h \leq$



(a)

(b)

(c)

Fig. 4. Simulation results for $r = 3$.

$h_{\min,2}$, the magnitude of $H_l(j\omega)$ surpasses $\frac{1}{r}$, which means (14) does not hold. In particular, when $r = 1$, the platoon is stable but not string stable. This agrees with Theorem 1 and Theorem 2. As shown in Fig. 3(c) and Fig. 4(c), when $h > h_{\min,2}$, the magnitude of $H_l(j\omega)$ does not surpass $\frac{1}{r}$. In this case, the platoons are both asymptotically stable and string stable, which confirms Theorem 1 and Theorem 2.

VI. Conclusion

This paper has explicitly addressed the approach to reduce time headway for string stable platoons via MPF. We have proposed a new definition of desired inter-vehicle distances using the CTH policy, and then derived a sufficient condition for string stability. It is proved that there exist feedback gains to ensure string stability if the time headway is lower bounded. This further indicates that increasing the number of predecessors can reduce the bound of time headway, which in turn improves transport capacity. Future work will investigate the effect of heterogeneity on the string stability.

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