

# Integrating Autonomy into Traffic Systems: Scalable Control and Optimization

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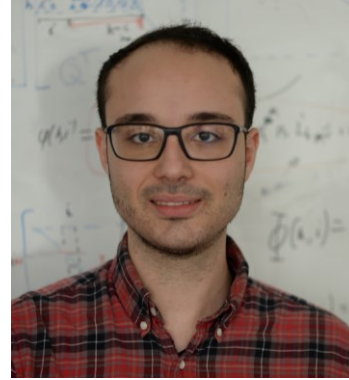
# Collaborators



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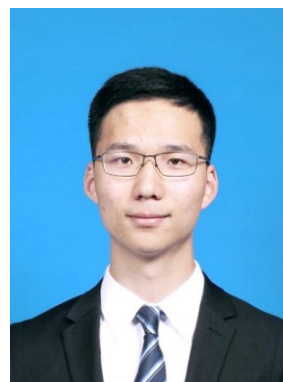
Luca Furieri  
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Maryam Kamgarpour  
ETH, Zurich



Keqiang Li  
Tsinghua University



Jiawei Wang  
Tsinghua University

# Autonomous Vehicles

- **Reduce traffic accidents**

- 37,000 fatalities
- 41% deaths of young adults (ages 15-24)
- **94%** of serious crashes caused by human error

- **Ease traffic congestion**

- **6.9 billion hours** wasted annually
- Cost of traffic congestion is **\$1740** per person annually in US/Europe.

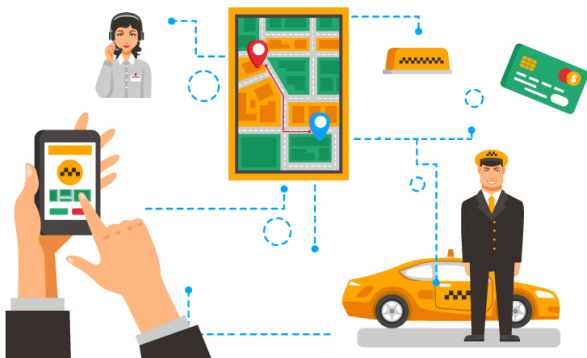
- **Improve energy efficiency**

- 28% of greenhouse gas emission is from transportation

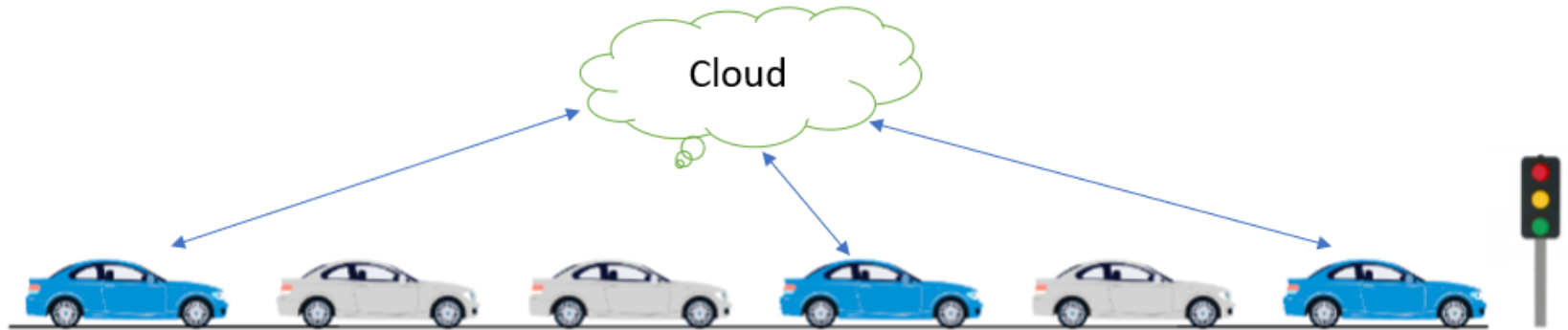
- **New mobility patterns:** on-demand mobility, mobility as service etc.



U.S. Census Bureau, 2017.



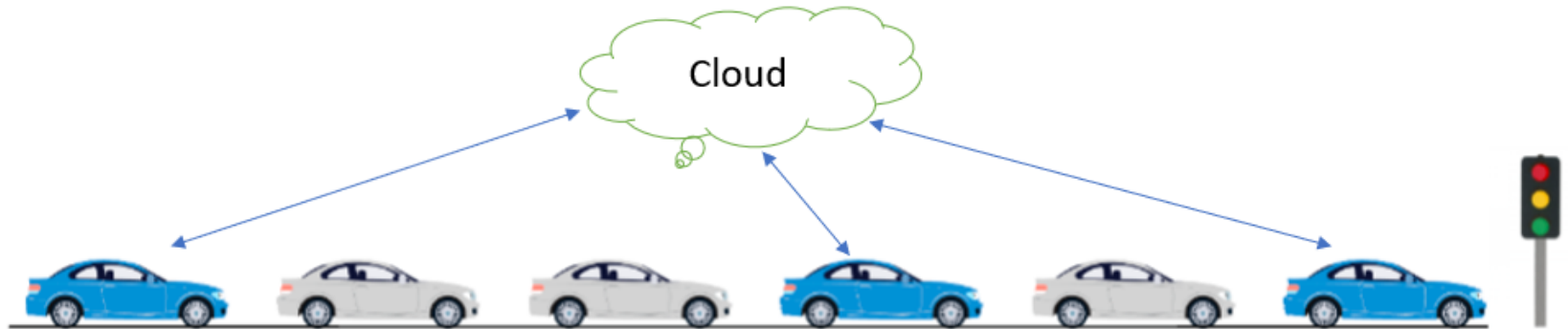
# Mix-Autonomy Mobility



**Mixed-autonomy mobility:** a traffic condition where both autonomous vehicles and human-driven vehicles co-exist.

- **Q1:** How will **a small scale of autonomous vehicles** change traffic dynamics?
- **Q2:** How to integrate **a small scale of autonomous vehicles** to improve traffic performance?

# Mix-Autonomy Mobility



- **Q1:** How will **a small scale of autonomous vehicles** change traffic dynamics?
- **Q2:** How to integrate **a small scale of autonomous vehicles** to improve traffic performance?

Theoretical evidence of  
the high potential of  
autonomous vehicles

Practical design via  
distributed control and  
scalable optimization

# Benchmark Ring Road Experiment

## Traffic jams



## Setting:

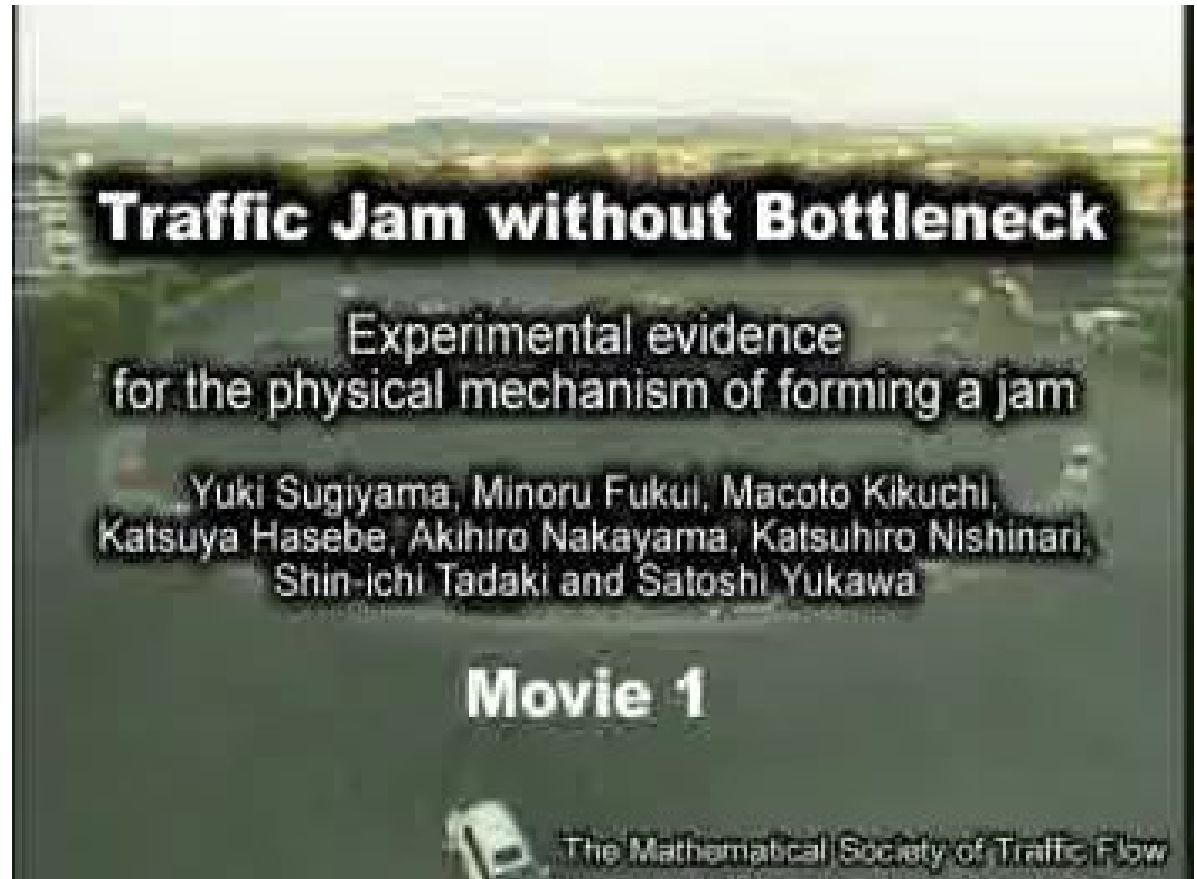
22 human drivers

## Instructions:

drive at 30 km/h  
/following its  
preceding vehicle

## Environment

Single lane  
No traffic lights,  
No stop signs,  
No lane changes.



# Benchmark Ring Road Experiment

## Traffic jams



## Setting:

21 human drivers

+ 1 AV

## Instructions:

drive at 30km/h  
/following its  
preceding vehicle

## Environment

Single lane  
No traffic lights,  
No stop signs,  
No lane changes.

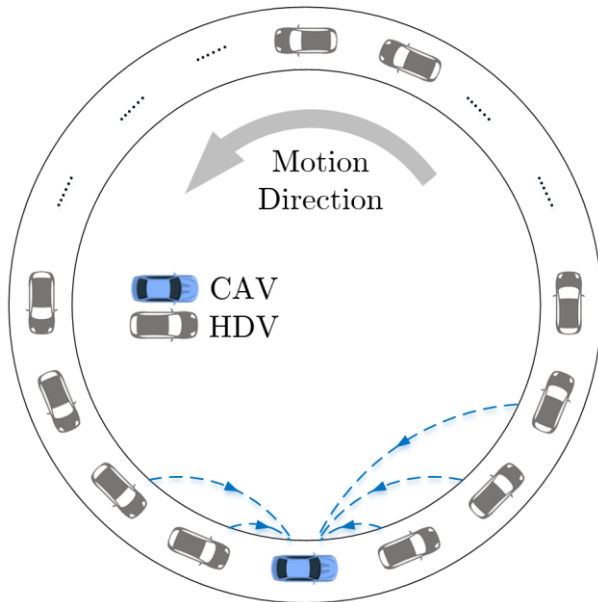
Dissipation of stop-and-go traffic waves via control of a single autonomous vehicle



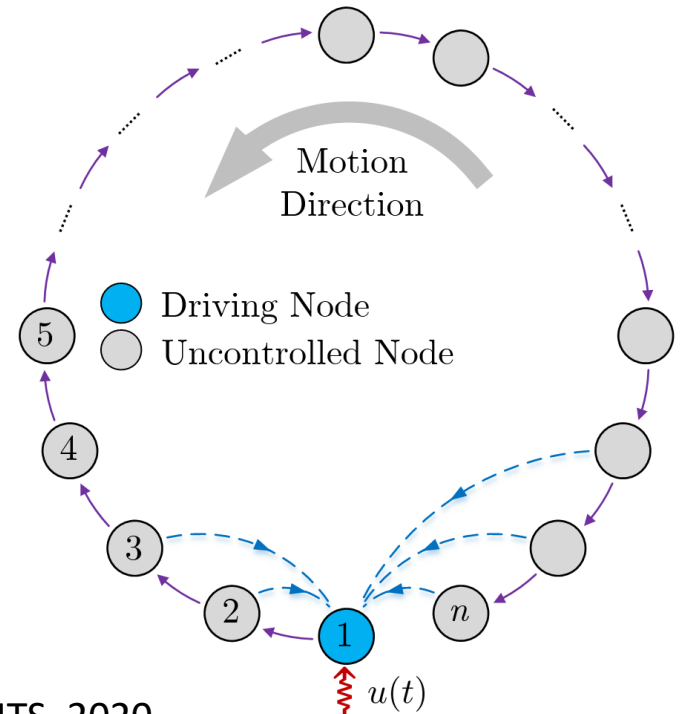
# Theoretical Evidence in mixed traffic

## □ Theoretical Evidence & Controller design

- Why does it work?
- Does it work in other setups (e.g., different number of HDVs, different human-driver behavior, open straight road scenario)?



**Sparse network control**

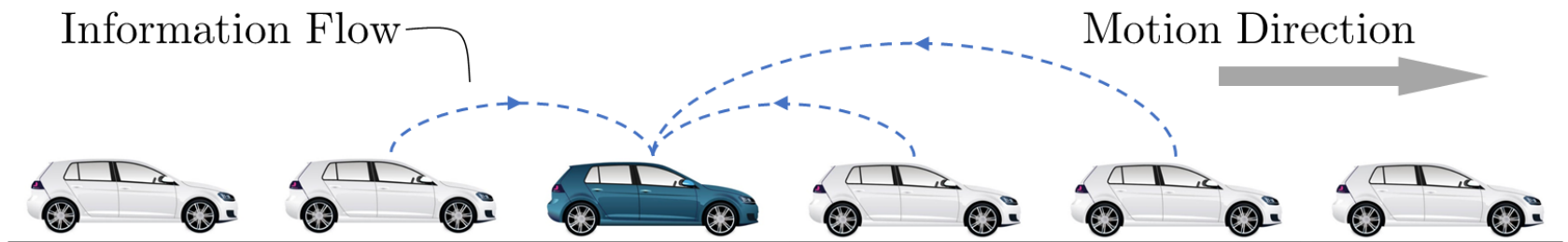




# Scalable Control & Optimization

## □ Theoretical Evidence & Controller design

- How to design distributed controllers with limited communication?
- How to scale up the computation efficiency?



- Furieri, L., **Zheng**, Y., Papachristodoulou, A., & Kamgarpour, M. (2020). Sparsity invariance for convex design of distributed controllers. *IEEE Transactions on Control of Network Systems*. (**Best Student Paper Finalist**, ECC 2019)
- **Zheng**, Y., Fantuzzi, G., Papachristodoulou, A., Goulart, P., & Wynn, A. (2020). Chordal decomposition in operator-splitting methods for sparse semidefinite programs. *Mathematical Programming*, 180(1), 489-532.

# Today's talk

## Integrating Autonomy into Traffic Systems

### Part 1: Theoretic potential of autonomy in traffic

- Stabilizability of mixed traffic flow;
- Autonomous vehicles as mobile actuators in traffic networks;
- Leading Cruise Control (LCC)

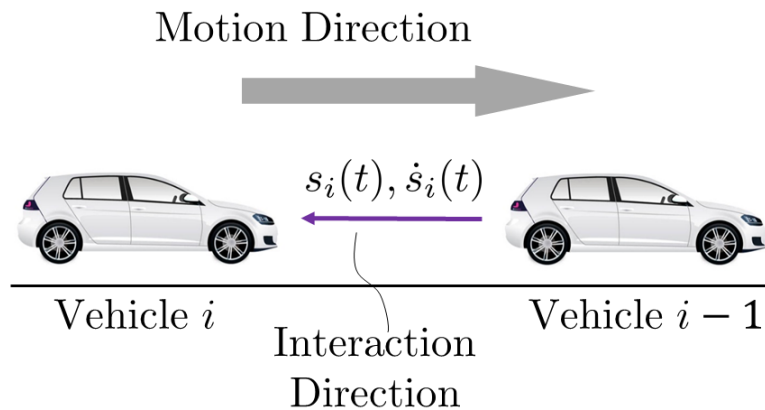
### Part 2: Practical design via control & optimization

- Convex design of distributed control over traffic network;
- Scalable optimization for large-scale convex problems;

# Mixed-autonomy ring road

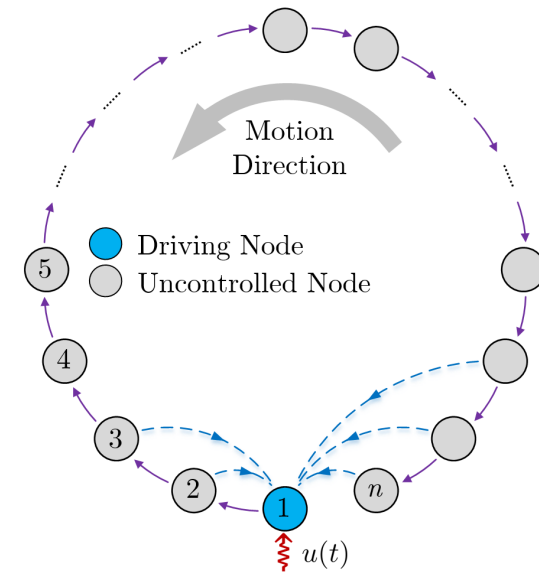
## □ System modeling

### 1. Human drivers → car-following dynamics

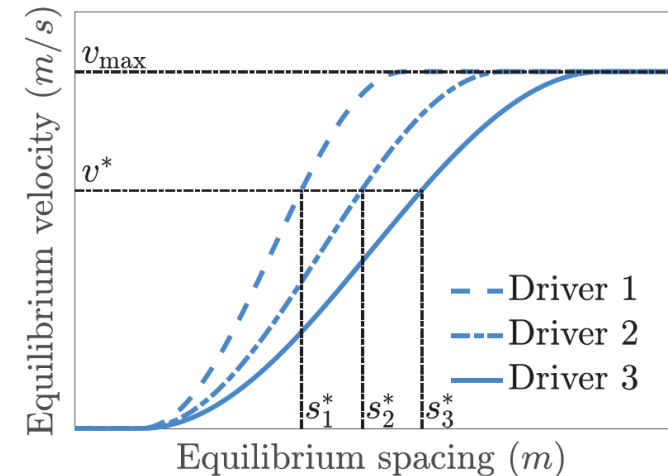


$$\dot{v}_i(t) = F_i(s_i(t), \dot{s}_i(t), v_i(t))$$

- $v_i(t)$ : Velocity of vehicle  $i$
- $s_i(t)$ : Spacing between vehicle  $i$  and vehicle  $i - 1$



$$0 = F_i(s_i(t), 0, v_i(t))$$



# Mixed-autonomy ring road

## □ System modeling

### 2. Autonomous vehicle → direct control

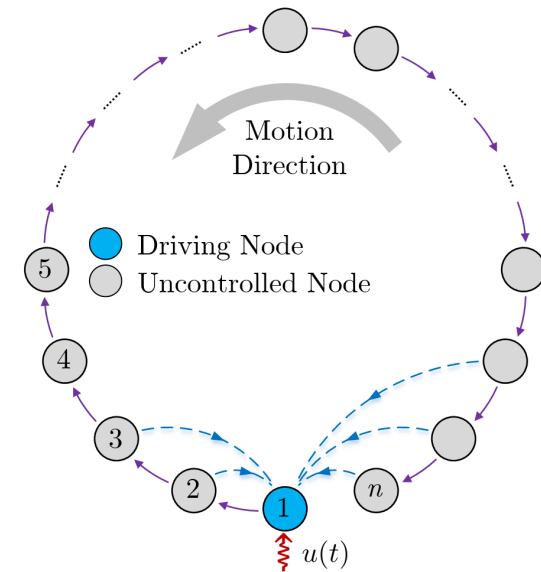
$$\begin{cases} \dot{s}_1(t) &= v_n(t) - v_1(t) \\ \dot{v}_1(t) &= u_1(t) \end{cases}$$

### 3. Assuming an equilibrium traffic state $v^*(t)$

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where the system matrices have the following structure

$$A = \begin{bmatrix} C_1 & 0 & \dots & \dots & 0 & C_2 \\ A_2 & A_1 & 0 & \dots & \dots & 0 \\ 0 & A_2 & A_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & A_2 & A_1 & 0 \\ 0 & \dots & \dots & 0 & A_2 & A_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$



A network system  
with only one  
controllable node

# Mixed-autonomy ring road

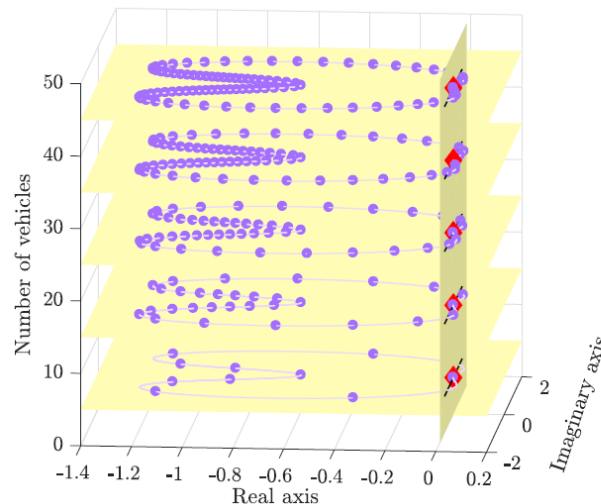
## □ Theoretical evidence 1: Unstable behavior

$$\dot{x}(t) = \hat{A}x(t)$$

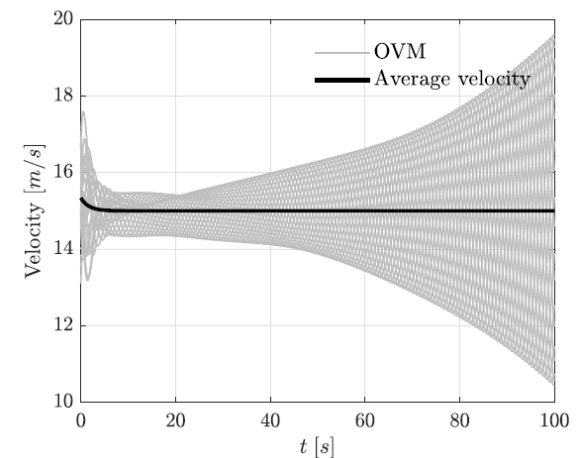
(Informal) The traffic system in a ring-road can be unstable if drivers' sensitivity to speed and spacing errors is small (e.g. Cui et al., 2017)

$$\alpha + 2\beta < \text{Constant}$$

Sensitivity to speed and spacing errors



Slow response to spacing; To catch up, it drives to a large velocity → **Oscillation**



# Mixed-autonomy ring road

## □ Theoretical evidence 2: Fundamental change of dynamics

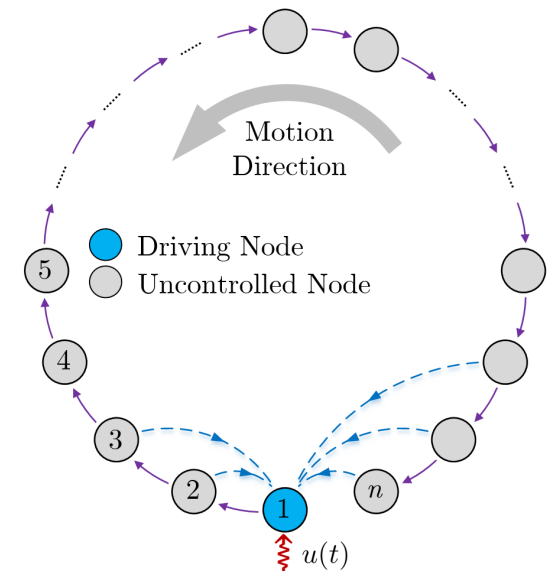
$$\dot{x}(t) = \hat{A}x(t)$$



**Theorem (zheng *et al.*, 2019): The mixed traffic system in the ring-road setup is not controllable, but stabilizable.**

1. Independent of the number of human-driven vehicles
2. Independent of car-following dynamics
3. Offer a strong control-theoretic support for the potential of autonomy in mixed traffic

Integrating autonomy is a fundamental change of traffic dynamics (more control freedom)!

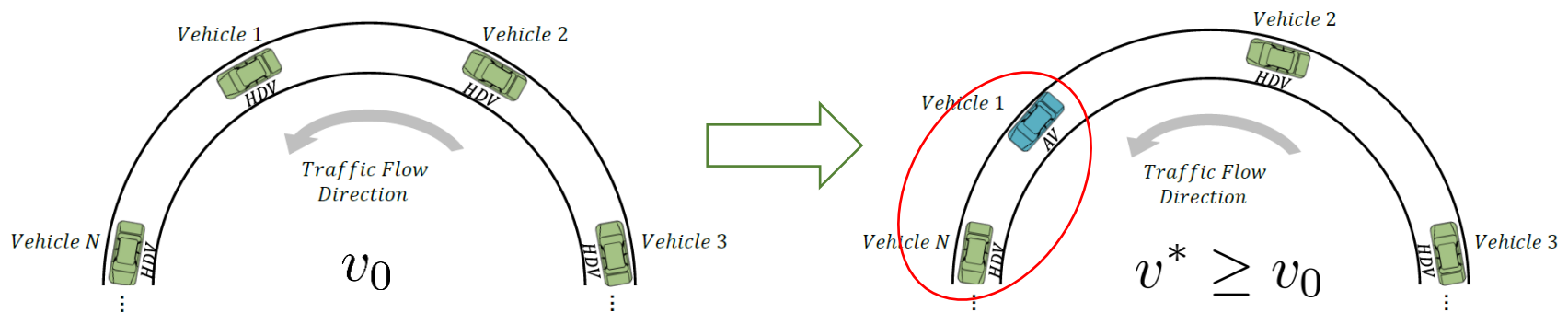


# Mixed-autonomy ring road

## □ Theoretical evidence 3: Beyond stabilization/increase traffic speed

**Theorem** (zheng *et al.*, 2019): **The global traffic velocity can be increased to a larger value:**

$$0 \leq v^* < v_{\max}$$

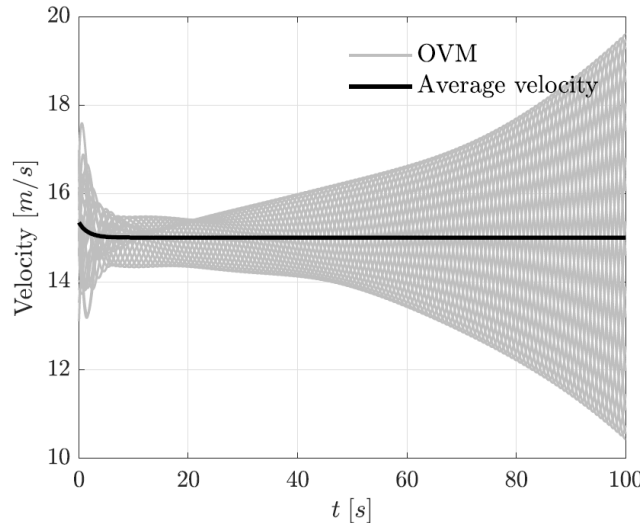


## □ Physical interpretation

- ✓ The AV can **change its own** spacing to **influence** other HDVs' spacing, and thus change traffic velocity  $v^*$ .

# Numerical Experiments with Nonlinear Dynamics

**Unstable** traffic system

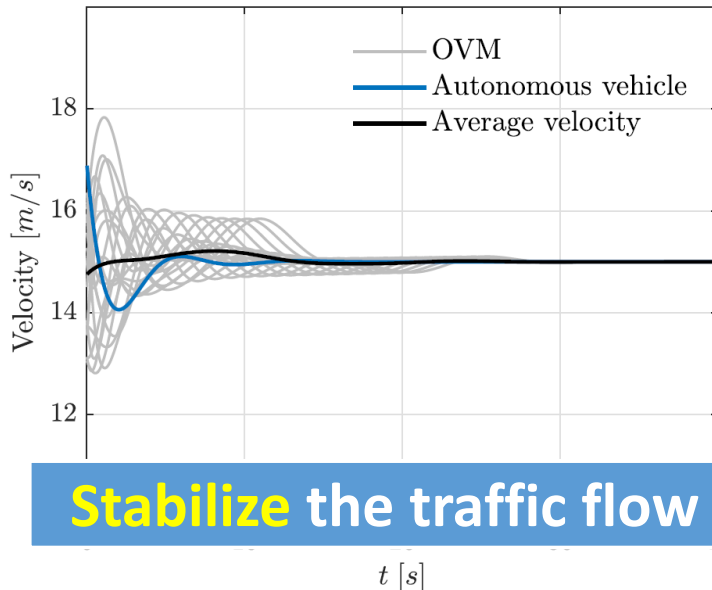


OVM: Optimal Velocity Model

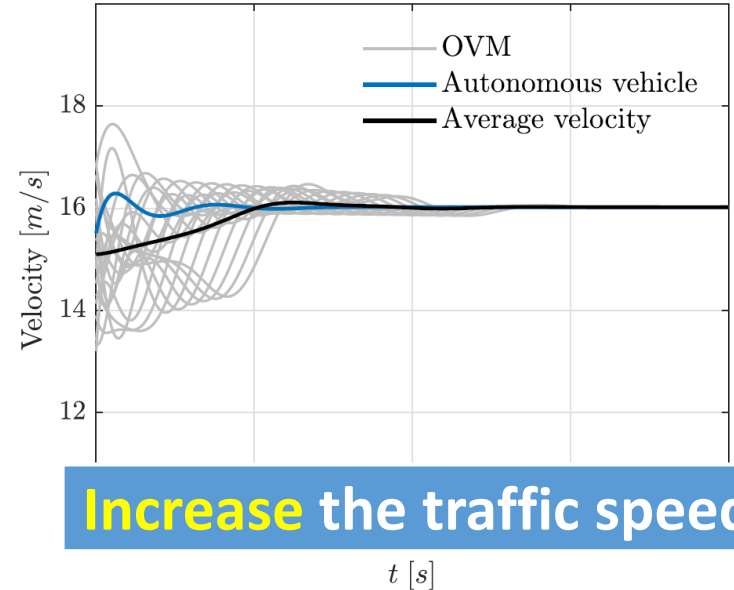
$$F_i = \alpha(V(s_i(t)) - v_i(t)) + \beta\dot{s}_i(t)$$

$$V(s) = \begin{cases} 0, & s \leq s_{st}, \\ f_v(s), & s_{st} < s < s_{go}, \\ v_{max}, & s \geq s_{go}, \end{cases}$$

$$f_v(s) = \frac{v_{max}}{2} \left( 1 - \cos\left(\pi \frac{s - s_{st}}{s_{go} - s_{st}}\right) \right).$$



**Stabilize** the traffic flow



**Increase** the traffic speed

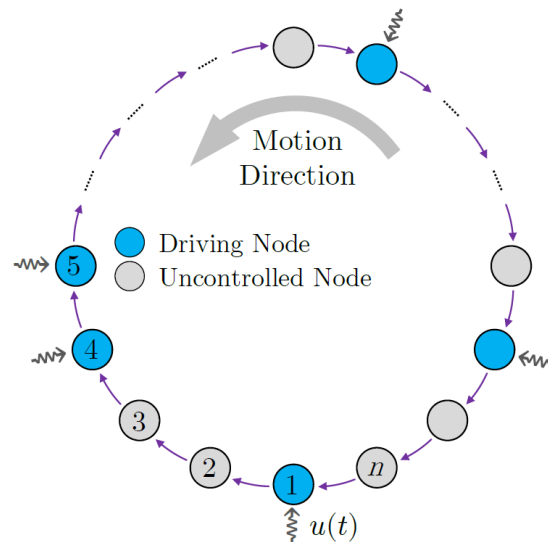
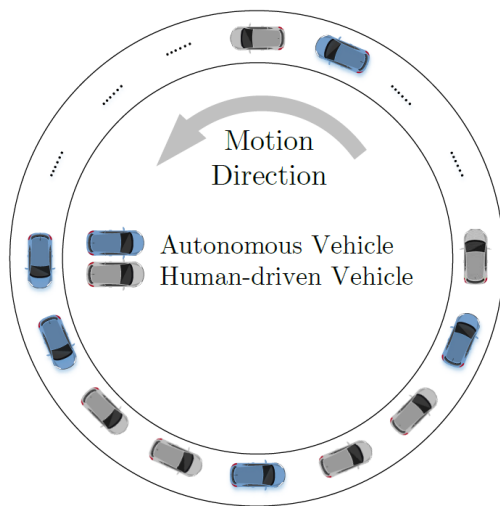
The existence of **5% AVs** (1 out of 20) can bring **6% improvement** on traffic velocity 16



# Integrating Autonomy: Multiple AVs



Main question: How to coordinate multiple autonomous vehicles in traffic flow? Is platooning the optimal choice?



Set function optimization

$$\max_S J(S)$$

$$S \subseteq \Omega, |S| = k$$

$\Omega = \{1, 2, \dots, n\}$ : all the vehicles

$S = \{i_1, \dots, i_k\} \subseteq \Omega$ :  $k$  autonomous vehicles

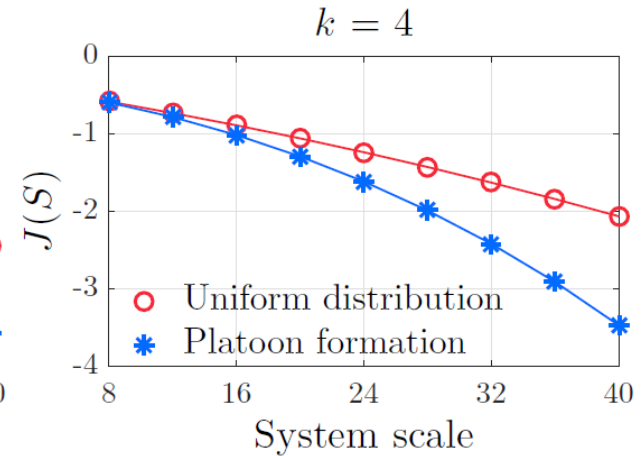
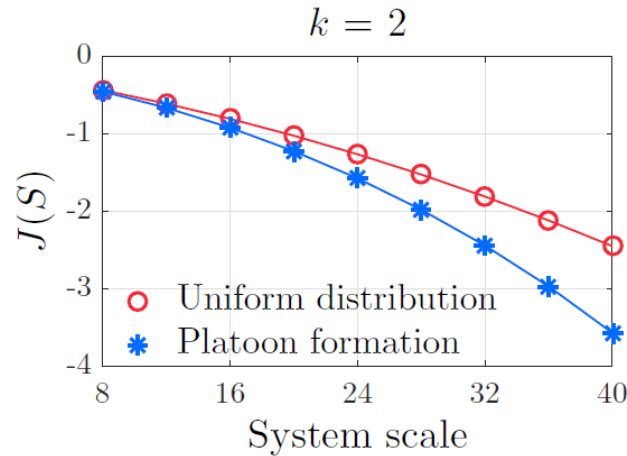
# Integrating Autonomy: Multiple AVs

Set function optimization

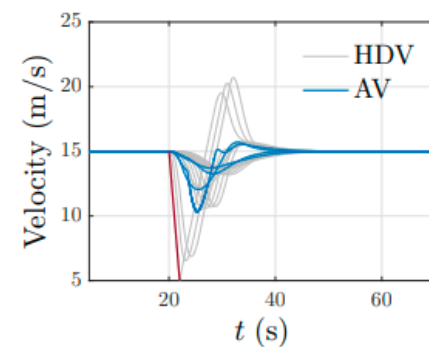
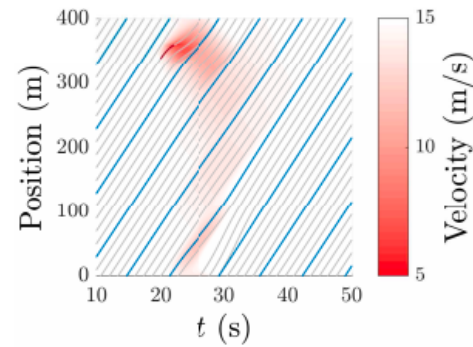
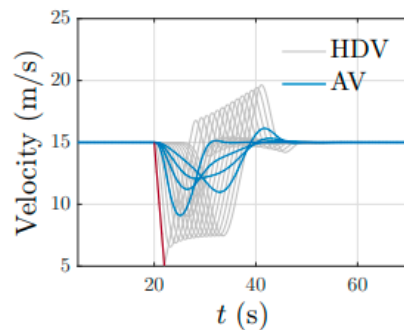
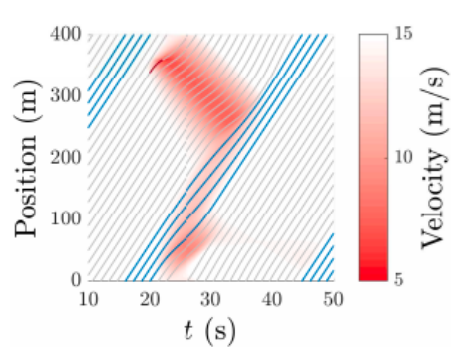
$$\max_S J(S)$$

$$S \subseteq \Omega, |S| = k$$

Platooning is not always optimal



## Simulation with Nonlinear Car-following Dynamics



Platoon formation:

$$S = \{9, 10, 11, 12\}$$

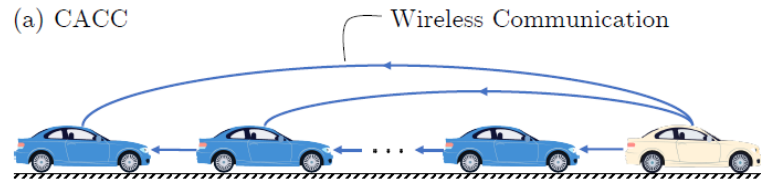
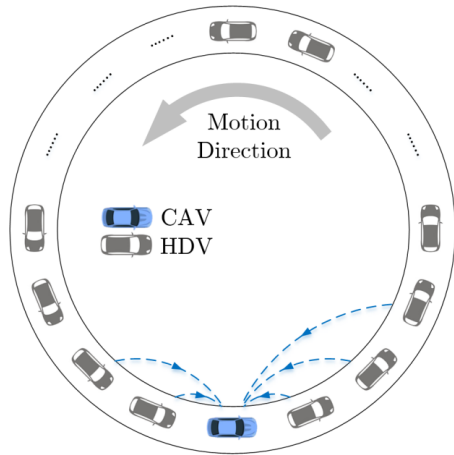
Li, Wang, & Zheng, (2020), IEEE TITS, under review

Uniform distribution:

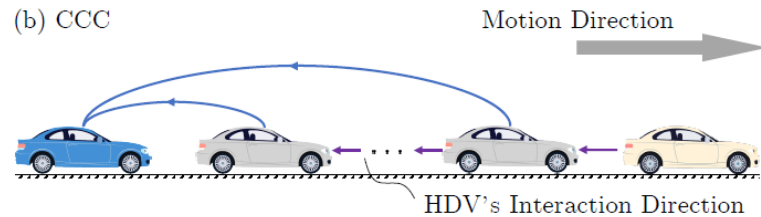
$$S = \{3, 8, 13, 18\}$$

# Integrating Autonomy in Open-straight road

## Closed-ring road setup

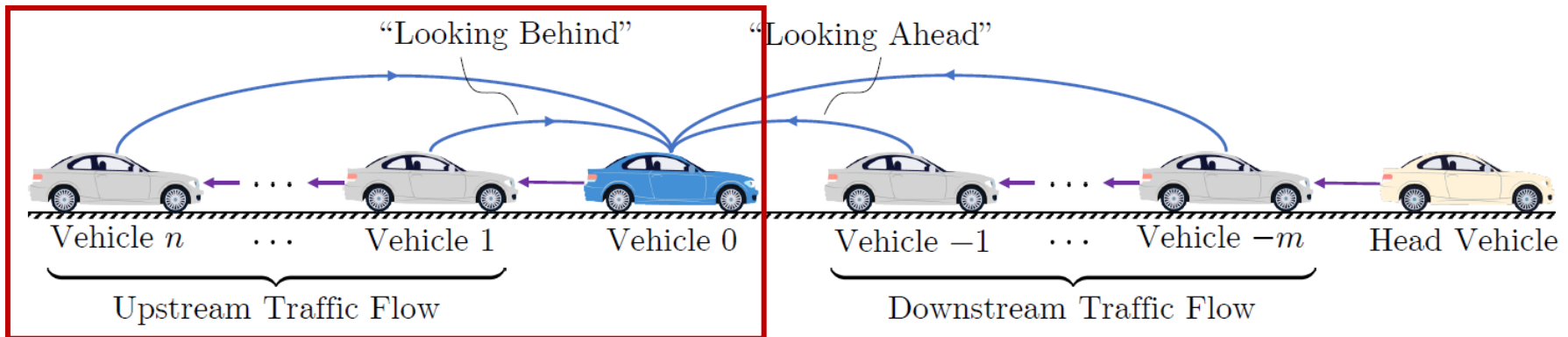


## CACC: Fully-autonomous scenario



## Connected Cruise Control: downstream traffic flow

## ➤ Leading Cruise Control



Lead the motion of the vehicles behind

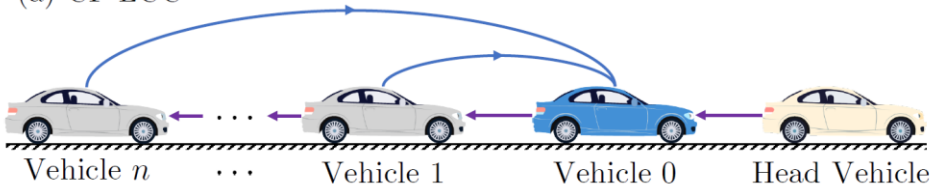


Adapt to the motion of the vehicles ahead

# Leading Cruise Control (LCC)

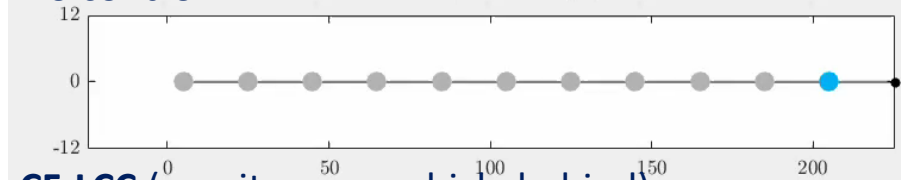
## Special case 1: car-following LCC

(a) CF-LCC

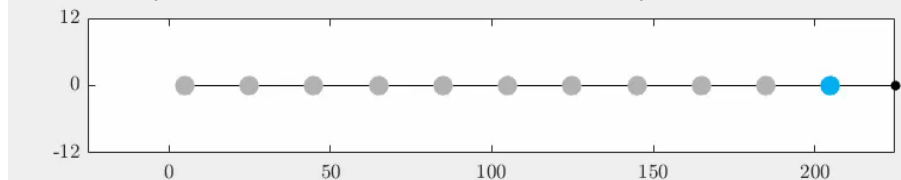


No control

Time = 15.0 s



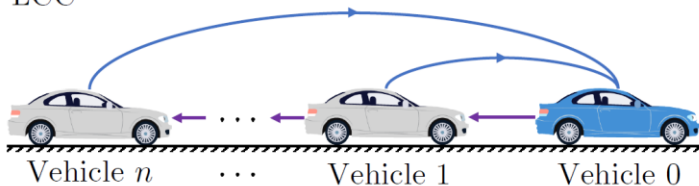
CF-LCC (monitor one vehicle behind)



Reduce velocity perturbations by **28%**

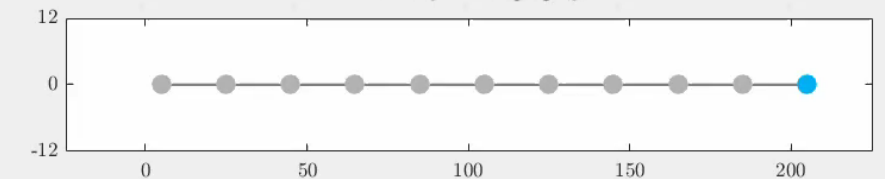
## Special case 2: free driving LCC

(b) FD-LCC

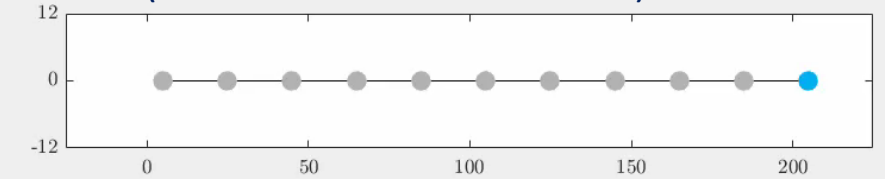


No control

Time = 15.0 s



FD-LCC (monitor one vehicle behind)



Reduce velocity perturbations by **35%**

1. The motion after AV is controllable (leading motion behind)
2. String stability can be improved (attenuating perturbation ahead)

# Today's talk

## Integrating Autonomy into Traffic Systems

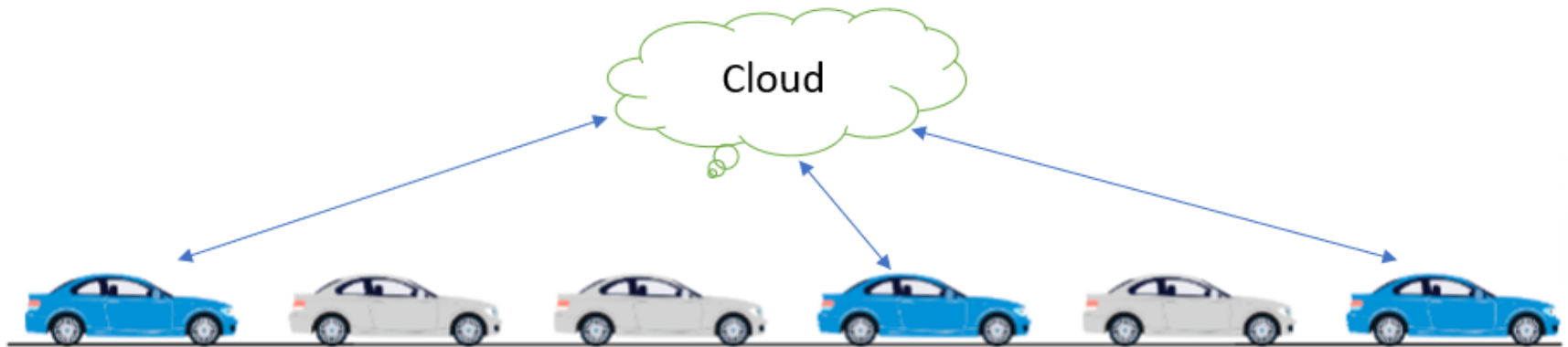
### Part 1: Theoretic potential of autonomy in traffic

- Stabilizability of mixed traffic flow;
- Autonomous vehicles as mobile actuators in traffic networks;
- Leading Cruise Control (LCC)

### Part 2: Practical design via control & optimization

- Convex design of distributed control over traffic network;
- Scalable optimization for large-scale convex problems;

# General Procedure



**Control Problem Formulation**



**Convex reformulation as LMI or SDP**

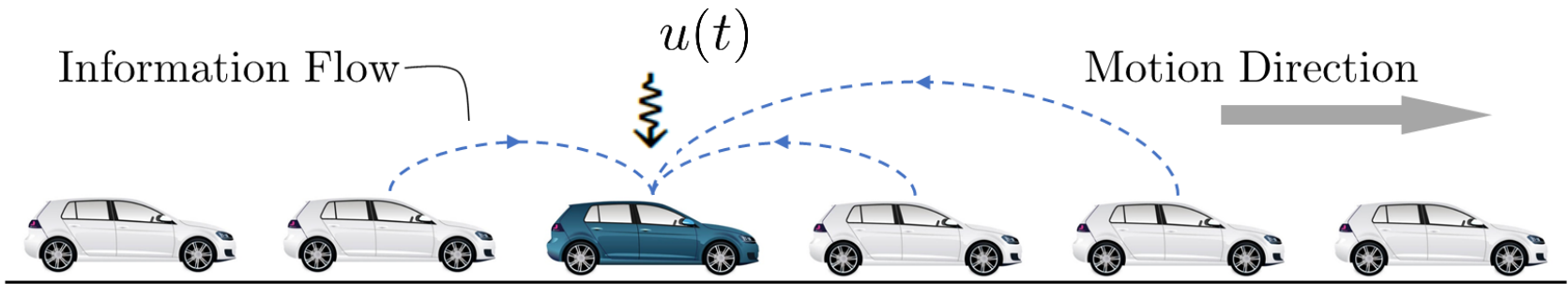


**Numerical solution**

**Challenge 1: How to handle info. constraints (recover convexity)**

**Challenge 2: How to deal with large-scale problems (Scalability)**

# Problem formulation: distributed controller



$$\begin{aligned} \min_K \quad & J(K) && \longrightarrow \text{System performance} \\ & && \text{(e.g., speed oscillation)} \\ \text{subject to} \quad & K \in \mathcal{C}_{\text{stab}}, && \longrightarrow \text{Stable controller} \\ & K \in \text{Sparse}(S) && \longrightarrow \text{Distributed controller} \end{aligned}$$

- This is a **non-convex optimization problem**
- The presence of the sparsity constraint makes the problem **challenging** (NP-hard in general).

# Previous work on distributed control

## □ 90's: Feasibility & Stabilization

- 1) **Structural controllability:** Glover & Silverman, [TAC 1976](#); Wang & Davison, [TAC 1973](#); Davison, [Automatica 1977](#); Mayeda and Yamada, [SICON 1979](#), etc.
- 2) **Decentralized/distributed fixed mode:** Anderson & Clements, [TAC 1981](#); Sezer & Šiljak, [SCL 1981](#); Davison & Özgüner, [Automatica 1983](#); etc.
- 3) **Decentralized stabilization & pole placement:** Davison & Chang, [TAC 1995](#); Ravi et al, [TAC 1995](#)
- 4) **Early survey paper:** Sandell, Varaiya, Athans & Safonov, [TAC 1978](#).

## □ Late 90's- Now: Performance enhancement via optimization

- 1) **Exact solutions for special classes of systems:** Quadratic Invariance (Rotkowitz & Lall, [TAC 2005](#)); Partially ordered sets (Shah & Parrilo, [TAC 2013](#));
- 2) **Tractable convex approximation:** Dvijotham et al, [TCNS 2015](#); Fazelnia et al, [TAC 2016](#);
- 3) **Suboptimal solutions using iterative algorithms:** Fu, Fardad, & Jovanovic, [TAC 2011](#);
- 4) **Structure regularization and system-level synthesis:** Jovanović & Dhingra, [2016](#); Wang et al., [TAC 2019](#);

**Recover**  
**Convexity**

A new framework based on Sparsity Invariance  
for convex design of distributed control



# Unified framework for distributed control

$$\begin{aligned} X \in \text{Sparse}(R), Y \in \text{Sparse}(T) \\ \Rightarrow \\ K = YX^{-1} \in \text{Sparse}(S) \end{aligned}$$

**Recover**  
**Convexity**

A new framework based on Sparsity Invariance  
for convex design of distributed control

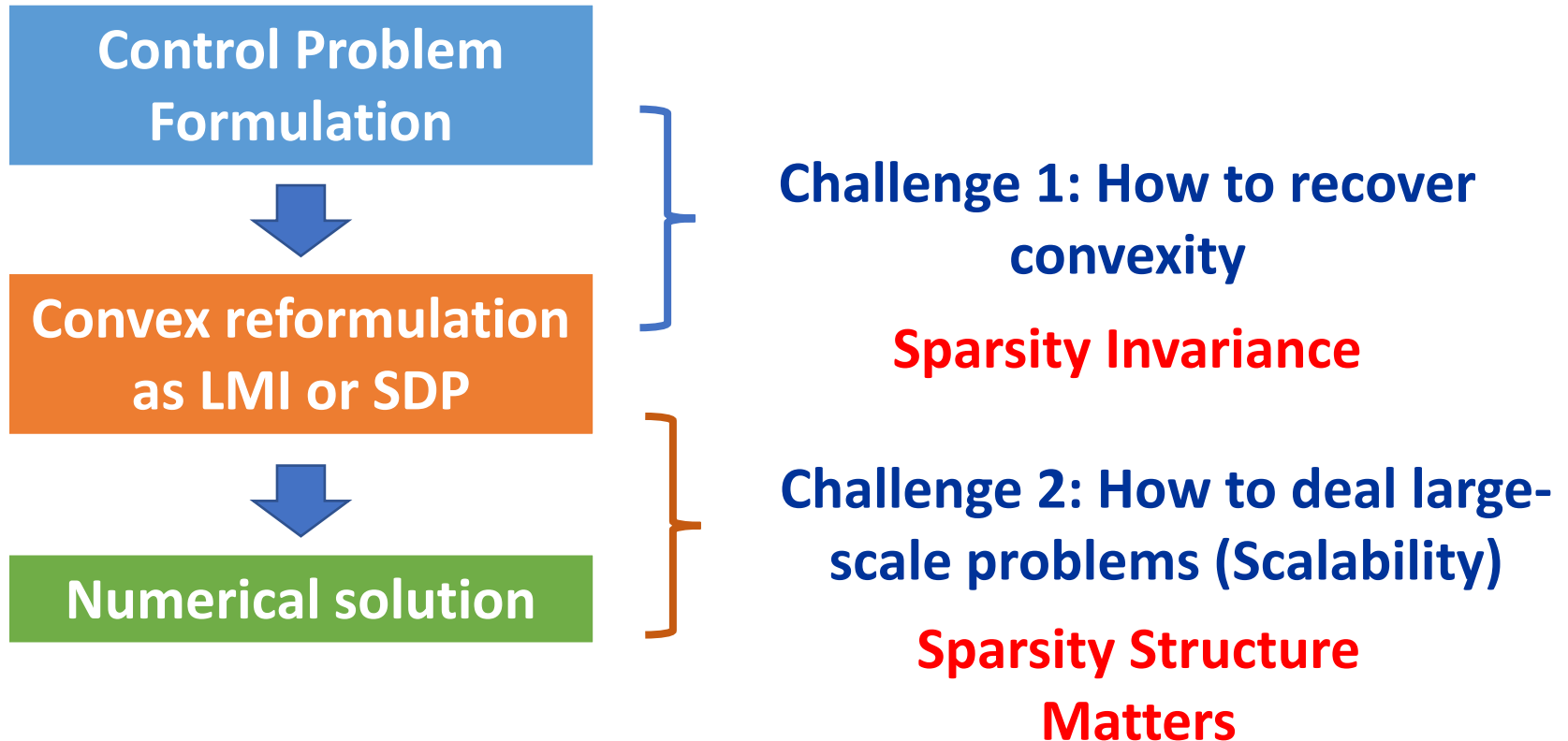
## Static feedback

- Strictly better than the widely used **diagonal approximation strategy** (Geromel et al., 1994; Conte et al., 2012; Rubio et al., 2013; Han et al., 2017)

## Dynamic feedback (past information + memory)

- Guaranteed to be optimal when a notion of **Quadratic Invariance (QI)** holds (Rotkowitz & Martins, 2012)
- Best known performance for non-QI cases

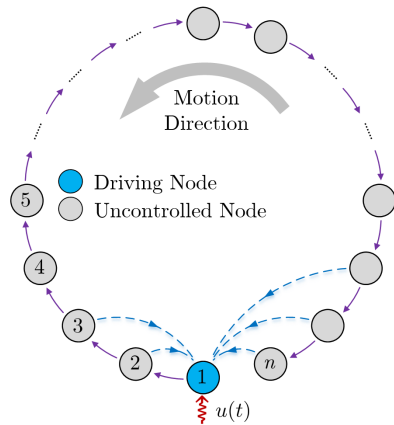
# General Procedure



# Sparsity Structure

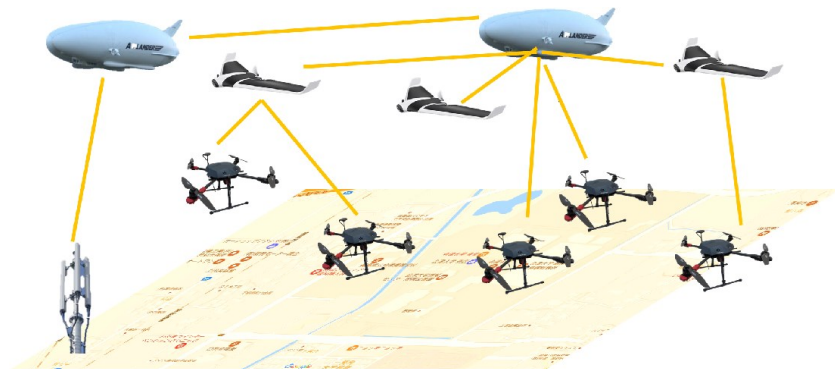
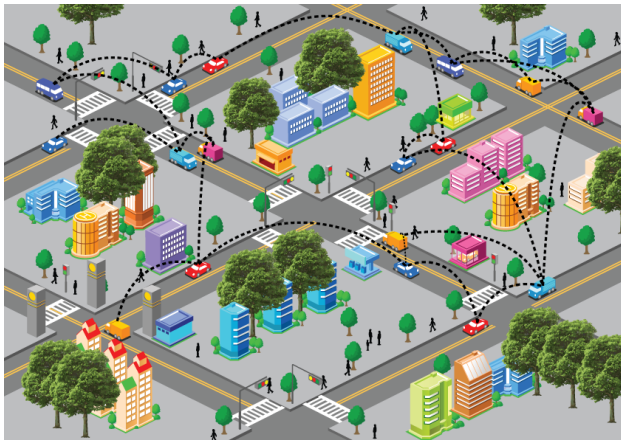
Sparsity structure appears in many places of real cyber-physical systems

## System dynamics data

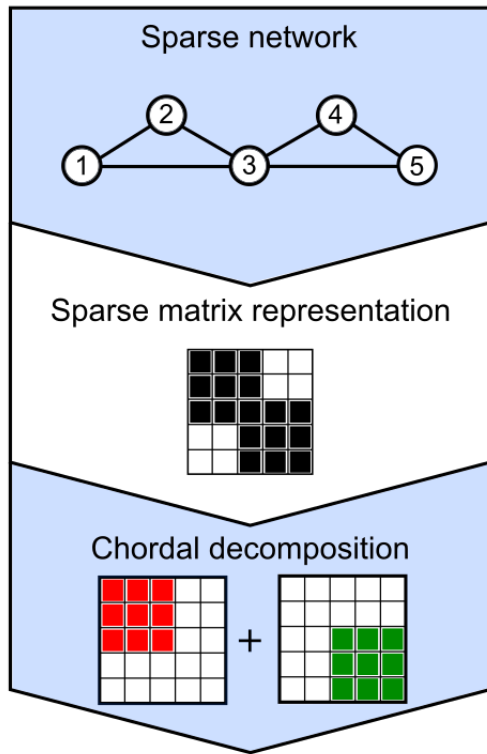


$$A = \begin{bmatrix} C_1 & 0 & \dots & \dots & 0 & C_2 \\ A_2 & A_1 & 0 & \dots & \dots & 0 \\ 0 & A_2 & A_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & A_2 & A_1 & 0 \\ 0 & \dots & \dots & 0 & A_2 & A_1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

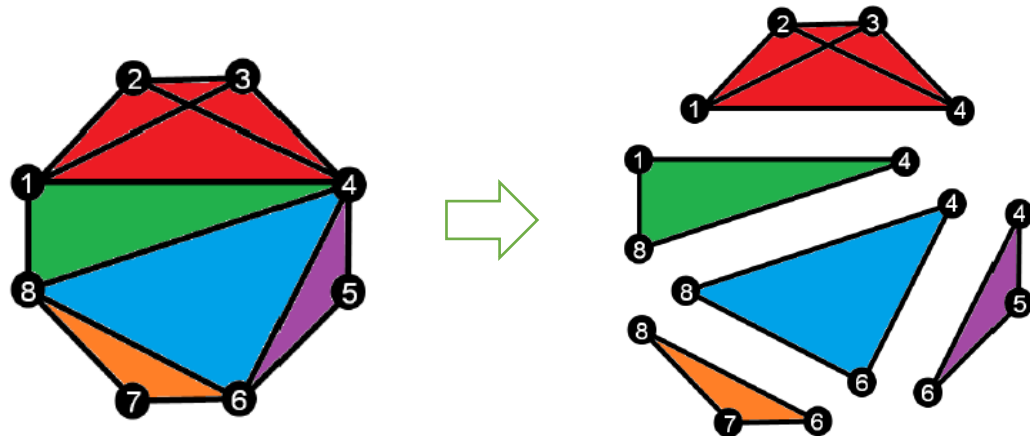
## Sparse communication



# Graph Decomposition



## Chordal graph decomposition



Vandenberghé & Andersen (2015).

- This allows for the decomposition of a big positive semidefinite constraint
- Exploiting this decomposition  $\rightarrow$  a new scalable algorithm for sparse SDP (Zheng et al. *Math. Prog.*, 2020)

# Conclusion

# Two main takeaways

## Theoretic potential of autonomy in traffic

- Mixed traffic systems is always **stabilizable**;
- Autonomous vehicles can not only **smooth traffic wave**, but also guide traffic velocity to a **higher value**;
- Autonomous vehicles can change traffic dynamics fundamentally (**Leading Cruise Control**)

## Integrating Autonomy via Control and Optimization

- **Convexity** of distributed control: a new framework based on sparsity invariance
- **Scalability** of convex optimization: Sparsity-exploiting methods based on graph decomposition

# References

## Autonomous Vehicles in Mixed Traffic Flow

1. **Zheng, Y.**, Wang, J., & Li, K. (2019). Smoothing traffic flow via control of autonomous vehicles. *IEEE Internet of Things Journal*, to appear, 1-15.
2. Li, K., Wang, J., & **Zheng, Y.** (2020). Cooperative Formation of Autonomous Vehicles in Mixed Traffic Flow: Beyond Platooning. arXiv preprint arXiv:2009.04254.
3. Wang, J., **Zheng, Y.**, Xu, Q., Wang, J., & Li, K. (2020). Controllability Analysis and Optimal Control of Mixed Traffic Flow with Human-driven and Autonomous Vehicles. arXiv preprint arXiv:2002.02099.
4. Wang, J., **Zheng, Y.**, Chen, C., Xu, Q., & Li, K. (2020). Leading Cruise Control in Mixed Traffic Flow. arXiv preprint arXiv:2007.11753.

## Controller Design & Scalability

1. Furieri, L., **Zheng, Y.**, Papachristodoulou, A., & Kamgarpour, M. (2019). Sparsity Invariance for Convex Design of Distributed Controllers. *IEEE Transactions on Control of Network Systems*, conditionally accepted. 1-11. (**Best Student Paper Award Finalist**, conference version)
2. **Zheng, Y.**, Fantuzzi, G., Papachristodoulou, A., Goulart, P., & Wynn, A. (2020). Chordal decomposition in operator-splitting methods for sparse semidefinite programs. *Mathematical Programming*, 1-44.

**Thank you for your attention!**

More information; visit <https://zhengy09.github.io/>