Benign Nonconvex Landscapes in Optimal and Robust Control

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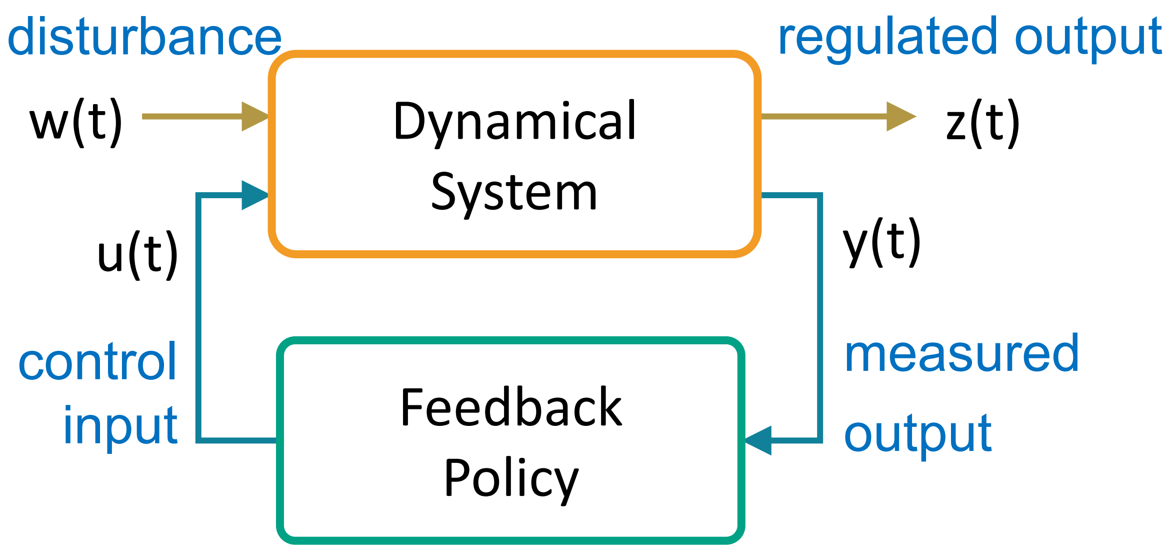
UC San Diego
JACOBS SCHOOL OF ENGINEERING
Electrical and Computer Engineering

Scalable Optimization
and Control (SOC) Lab
https://zhengy09.github.io/soclab.html

Policy Optimization in Control

❑ Optimal and Robust Control



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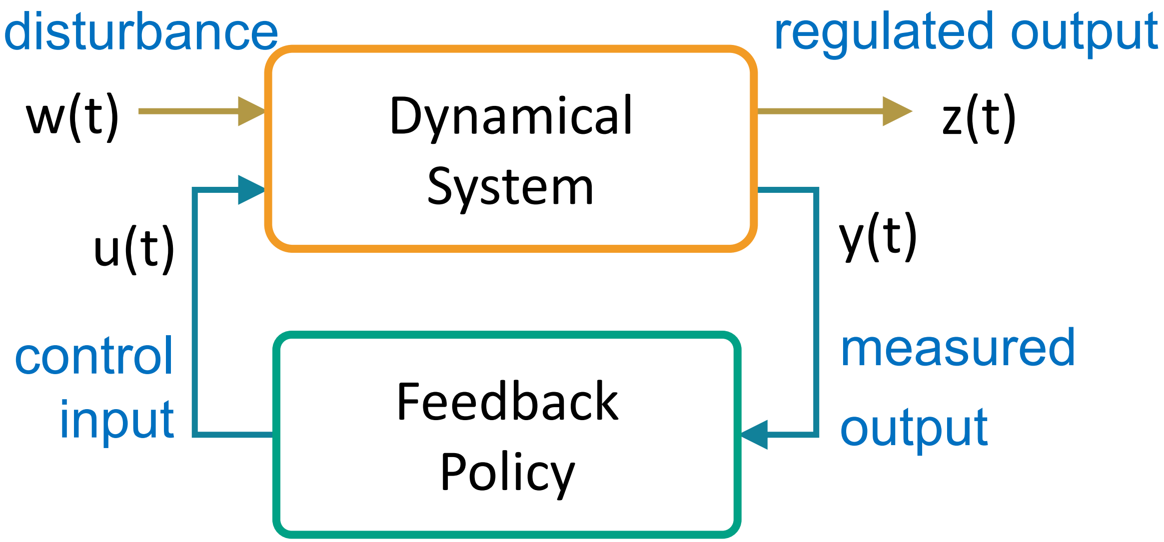
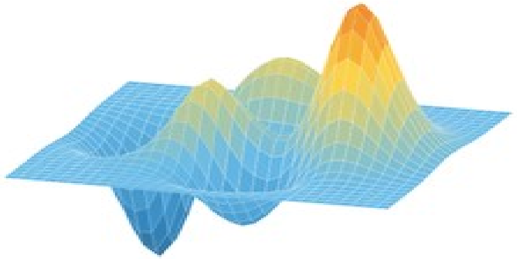


Policy parametrization



$$\min_{\mathsf{K}} \quad J(\mathsf{K}) \qquad \text{s.t.} \quad \mathsf{K} \in \mathcal{C}$$

Non-convex Optimization problem

Rich Geometry in Policy Optimization

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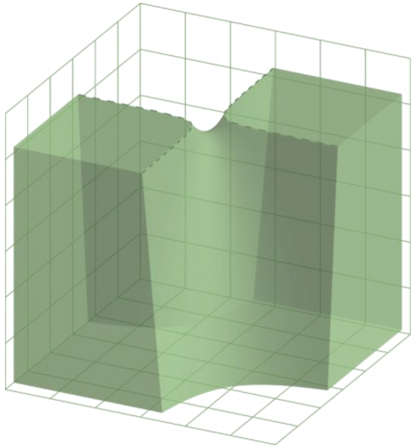

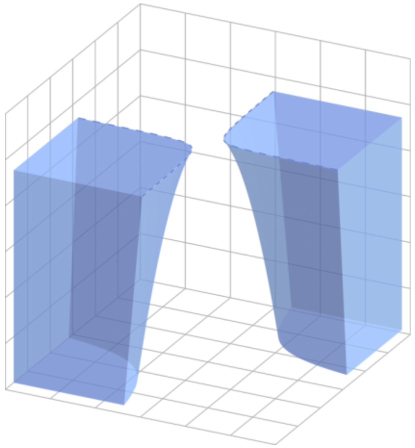

- The set of (dynamic) stabilizing policies is **nonconvex** and even might be **not connected.** [Tang, **Zheng**, Li, 2023]

Rich Geometry in Policy Optimization

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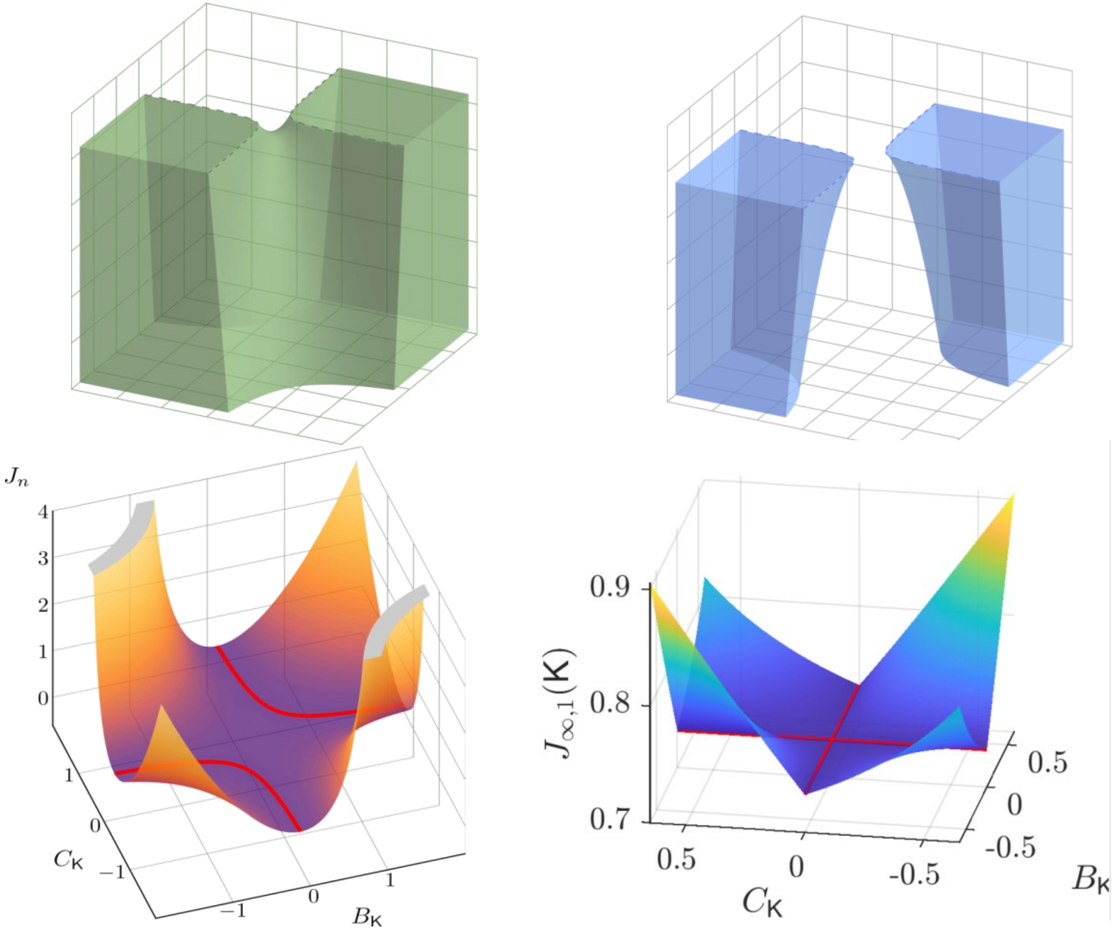
- LQR/LQG costs are **smooth but nonconvex**; the Hinf cost is **non-smooth and nonconvex**



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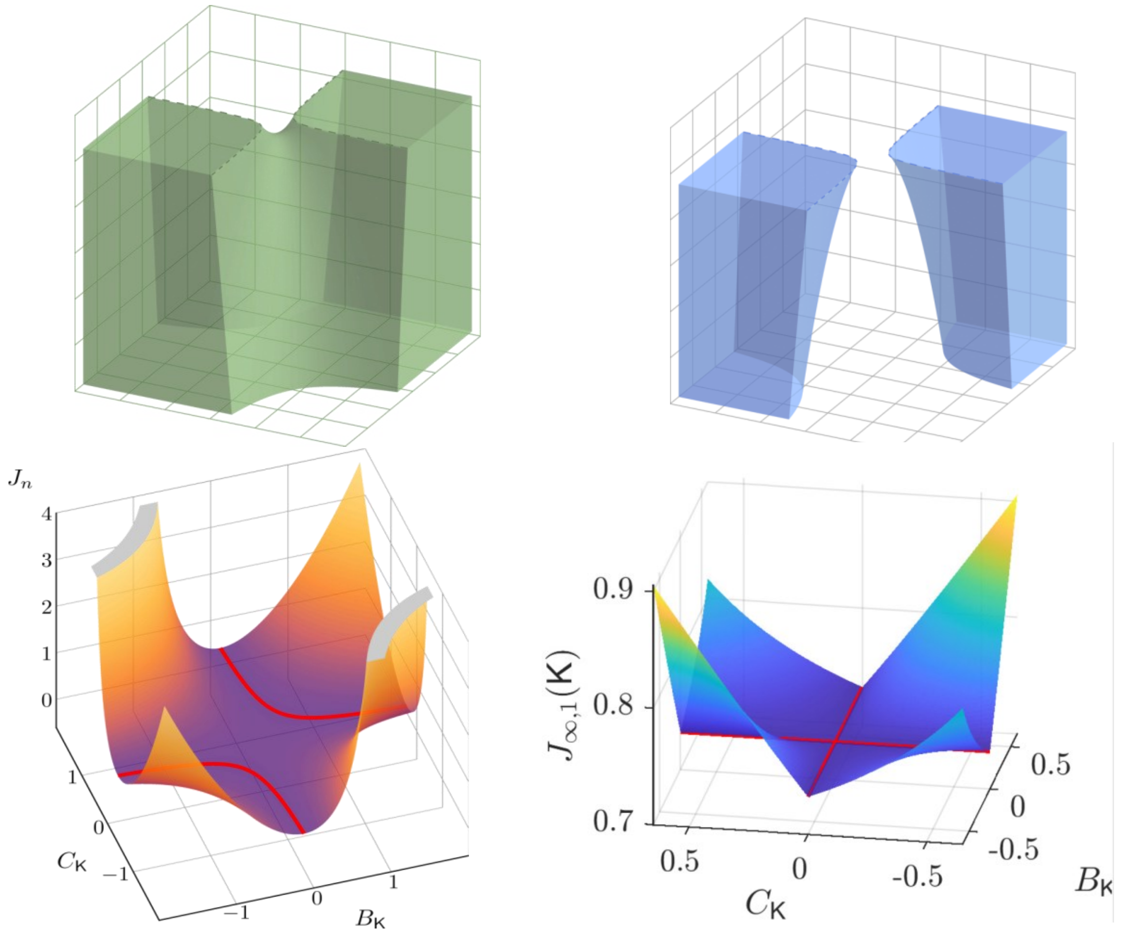

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For all iconic control problems (LQR, LQG, Hinf etc.), any (non-degenerate) **Clarke stationary points are globally optimal!**

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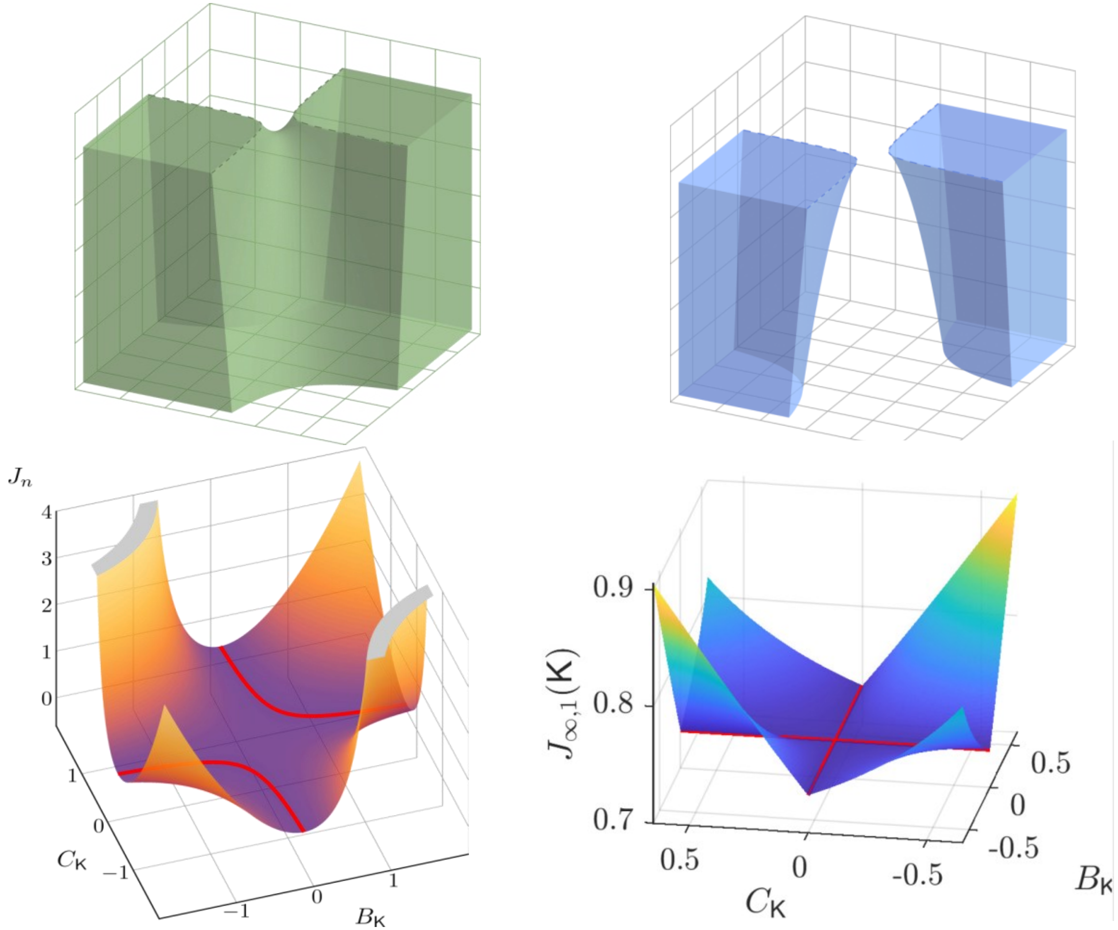

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Local Stationarity + **Structural Information**

Global Optimality Certificate

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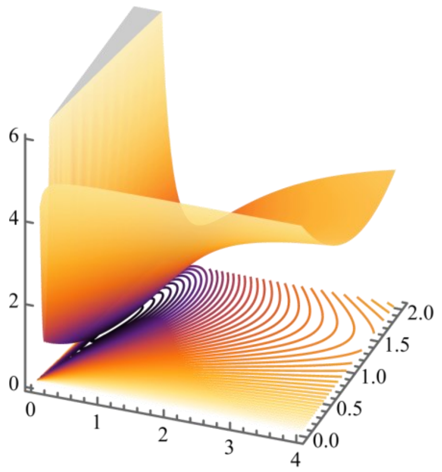
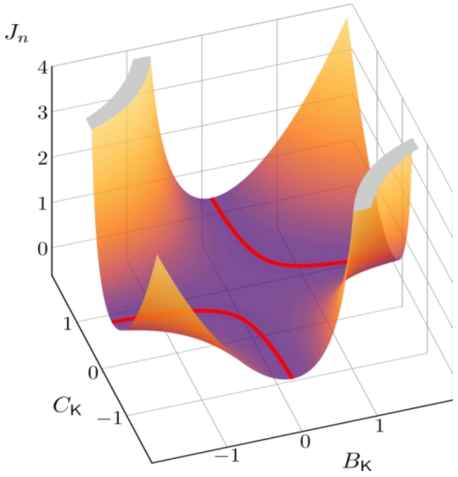
Our Technique: Extended Convex Lifting

Nonconvex Policy Optimization

(Modern Perspective)





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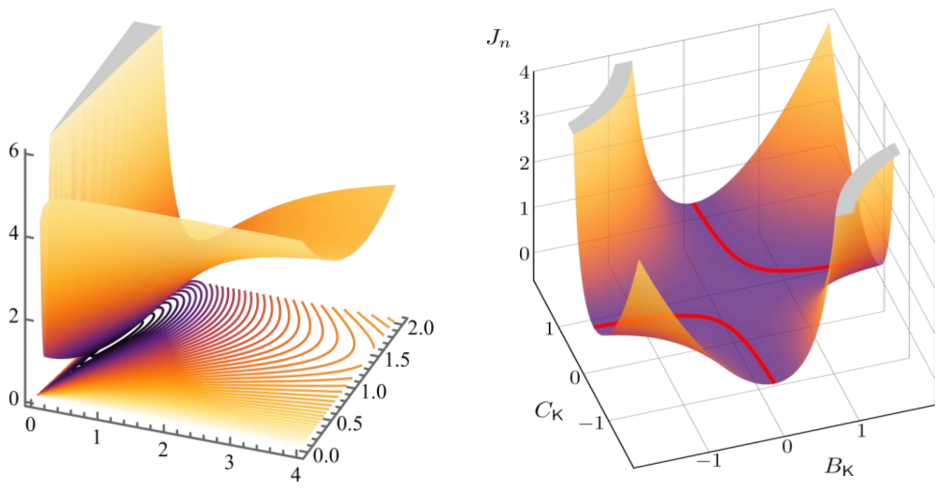
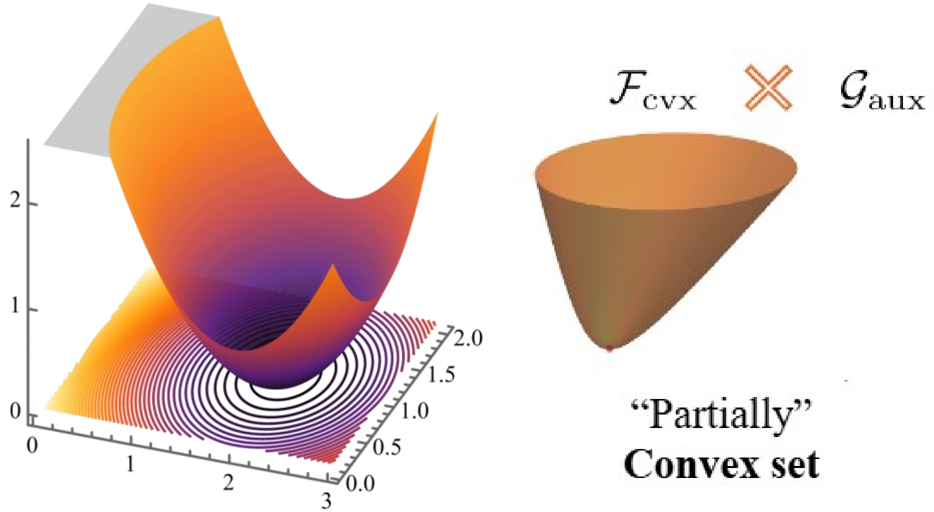


Convex LMIs
(Classical tool)

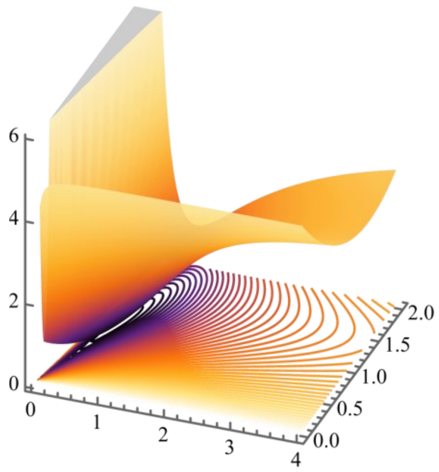


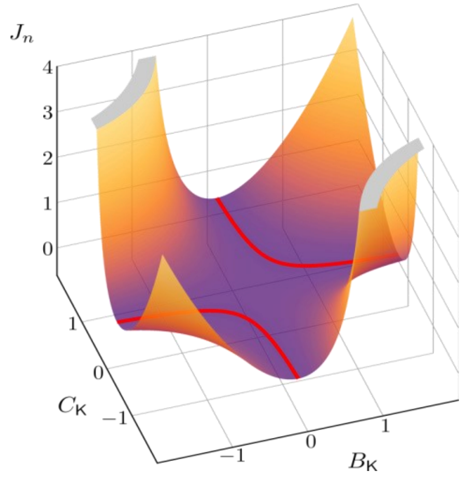
Our Technique: Extended Convex Lifting

Nonconvex Policy Optimization
(Modern Perspective)

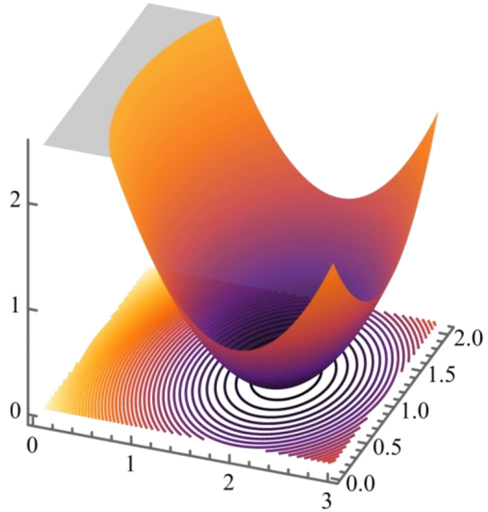
A framework of
Extended Convex Lifting (ECL)

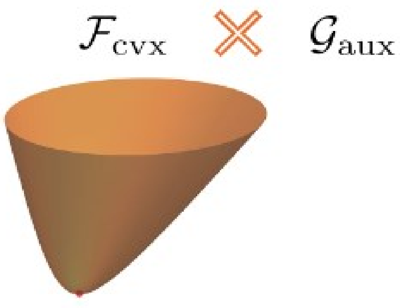
Convex LMIs
(Classical tool)





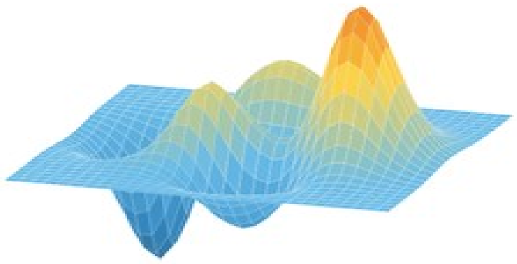
"Invertible" Map





❖ Reconciles the gap between **nonconvex policy optimization** and **convex reformulations**.

Lifting for Convexity



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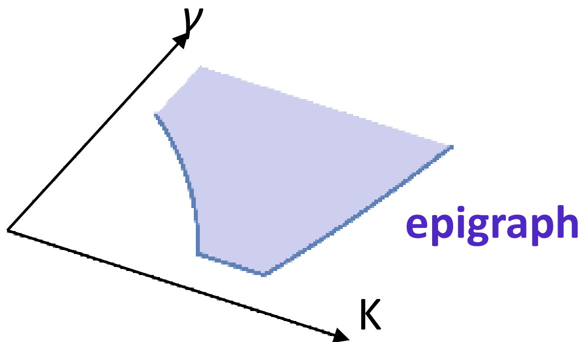
Non-convex Optimization problem

- For many control problems, a **direct convexification is not possible!**
- **A lifting procedure** corresponding to **Lyapunov variables** is necessary.



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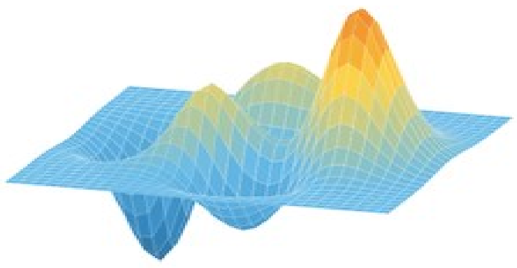

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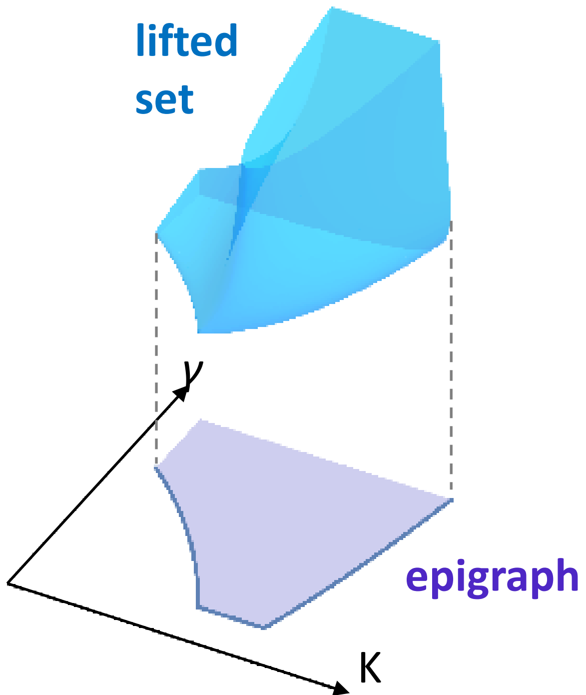

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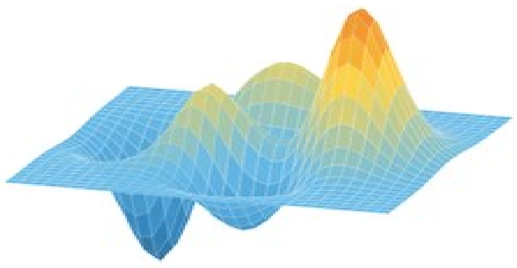

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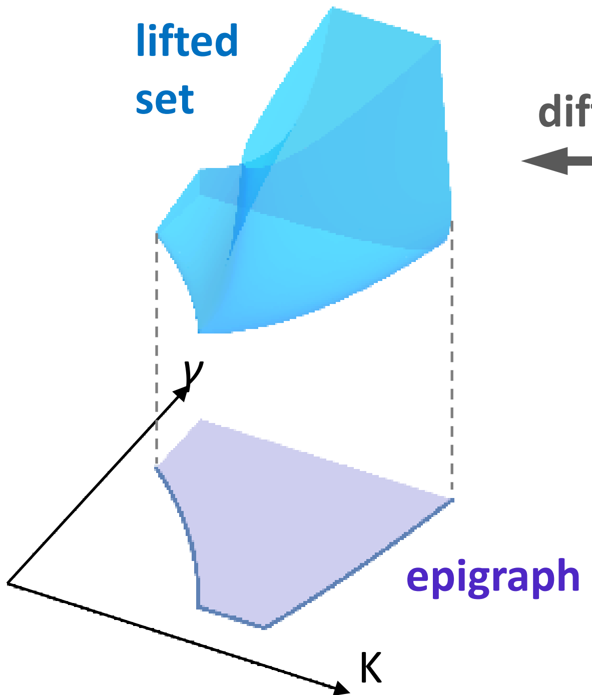
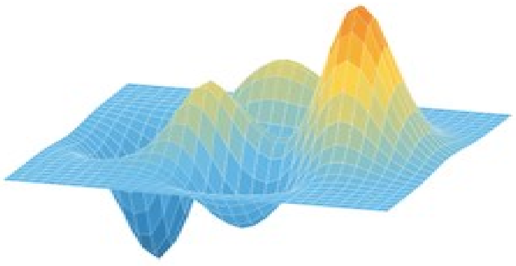


diffeomorphism ⟷ $\mathcal{F} \times \mathrm{GL}_n$

$\mathcal{F}$: convex

GL_n: set of invertible matrices

Lifting for Convexity



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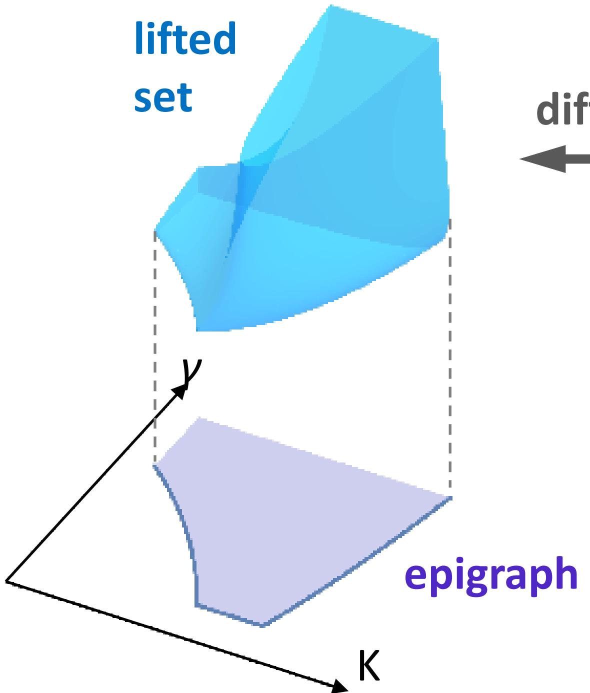

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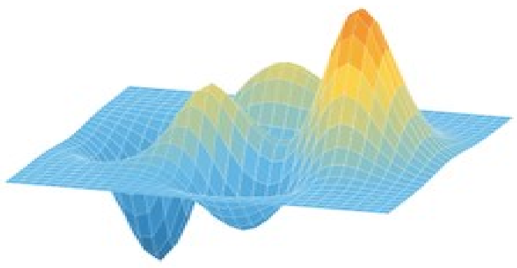
convex ⇓ **Global Optimality Certificate** ⇐ **Local Stationarity** + **Structural Information**

Lifting for Convexity



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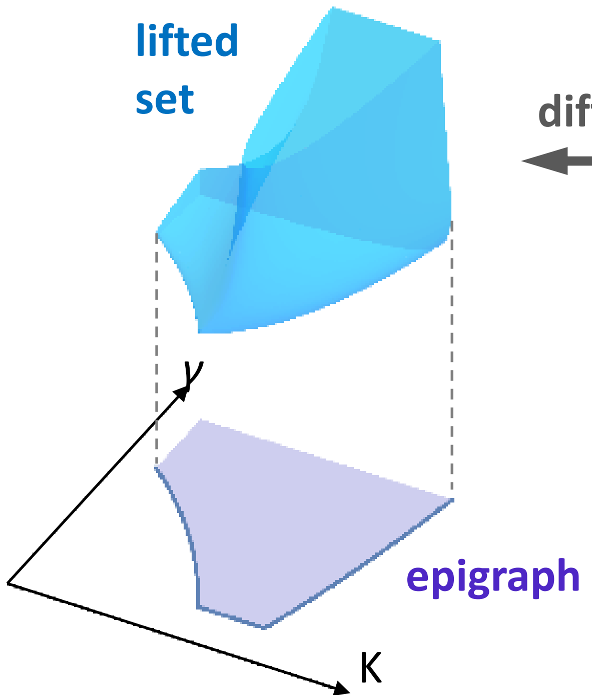

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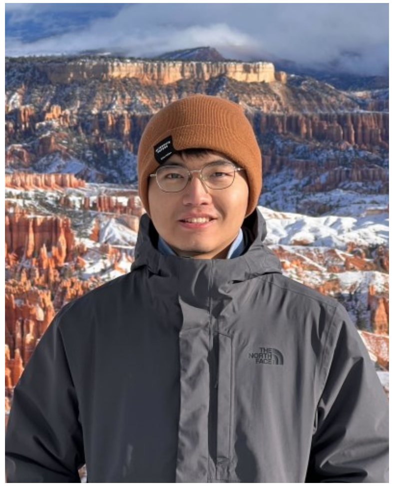
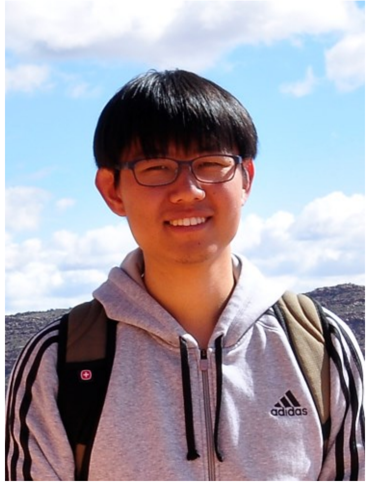
See our poster for more details!

Acknowledgements



Chih-Fan (Rich) Pai
University of California San Diego



Yujie Tang
Peking University

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- Zheng, Yang, Chih-Fan Pai, and Yujie Tang. "**Benign Nonconvex Landscapes in Optimal and Robust Control, Part I: Global Optimality**." arXiv preprint arXiv:2312.15332 (2023): https://arxiv.org/abs/2312.15332.
- Zheng, Yang, Chih-Fan Pai, and Yujie Tang. "**Benign Nonconvex Landscapes in Optimal and Robust Control, Part II: Extended Convex Lifting**." arXiv preprint arXiv:2406.04001 (2024): https://arxiv.org/abs/2406.04001