

# Integrating Autonomy into Traffic Systems: Scalable Control and Optimization

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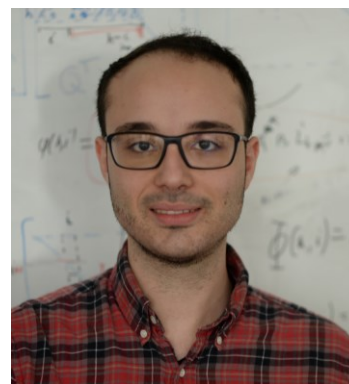
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EPFL

# Autonomous Vehicles

- **Reduce traffic accidents**

- 37,000 fatalities
- 41% deaths of young adults (ages 15-24)
- **94%** of serious crashes caused by human error

- **Ease traffic congestion**

- **6.9 billion hours** wasted annually
- Cost of traffic congestion is **\$1740** per person annually in US/Europe.

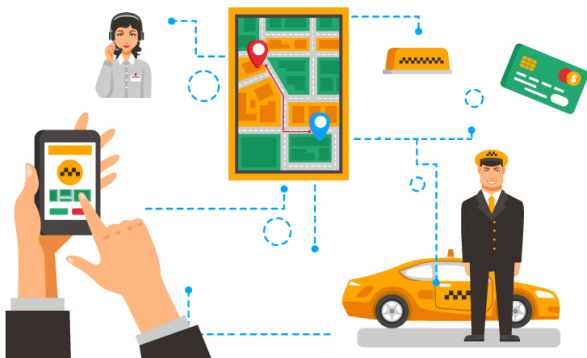
- **Improve energy efficiency**

- 28% of greenhouse gas emission is from transportation

- **New mobility patterns:** on-demand mobility, mobility as service etc.

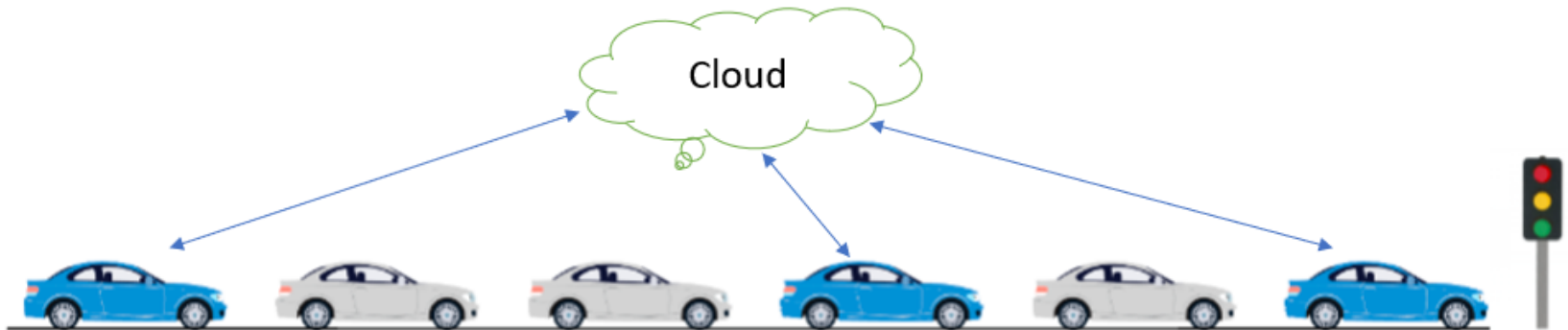


U.S. Census Bureau, 2017.



# Mix-Autonomy Mobility

## A long stage of mixed-autonomy mobility

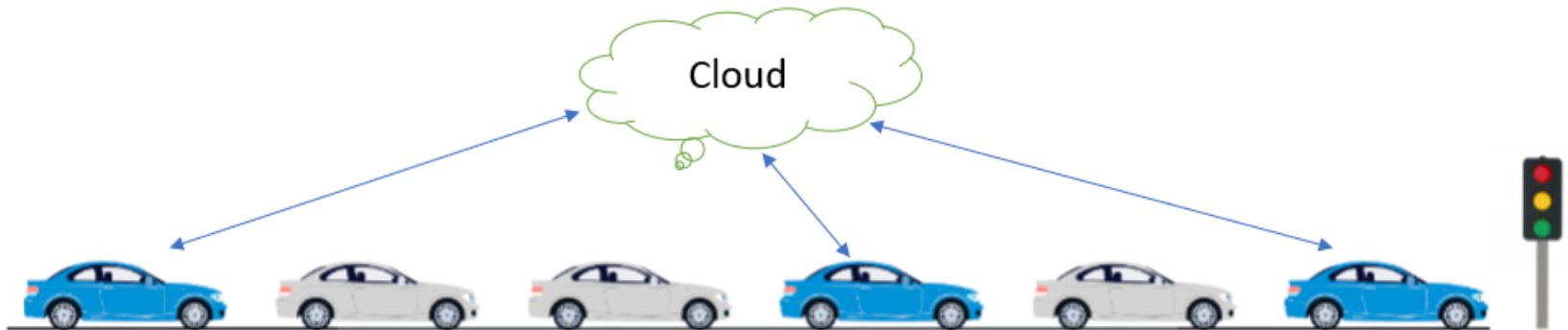


**Mixed-autonomy mobility:** a traffic condition where both autonomous vehicles and human-driven vehicles co-exist.

- **Q1:** How will **a small scale of autonomous vehicles** change traffic dynamics?
- **Q2:** How to integrate **a small scale of autonomous vehicles** to improve traffic performance?

# Research questions

## Mixed-autonomy mobility



- **Q1:** How will **a small scale of autonomous vehicles** change traffic dynamics?
- **Q2:** How to integrate **a small scale of autonomous vehicles** to improve traffic performance?

Theoretical evidence of  
the high potential of  
autonomous vehicles

Practical design via  
distributed control and  
scalable optimization

# Benchmark Ring Road Experiment

## Traffic jams



## Setting:

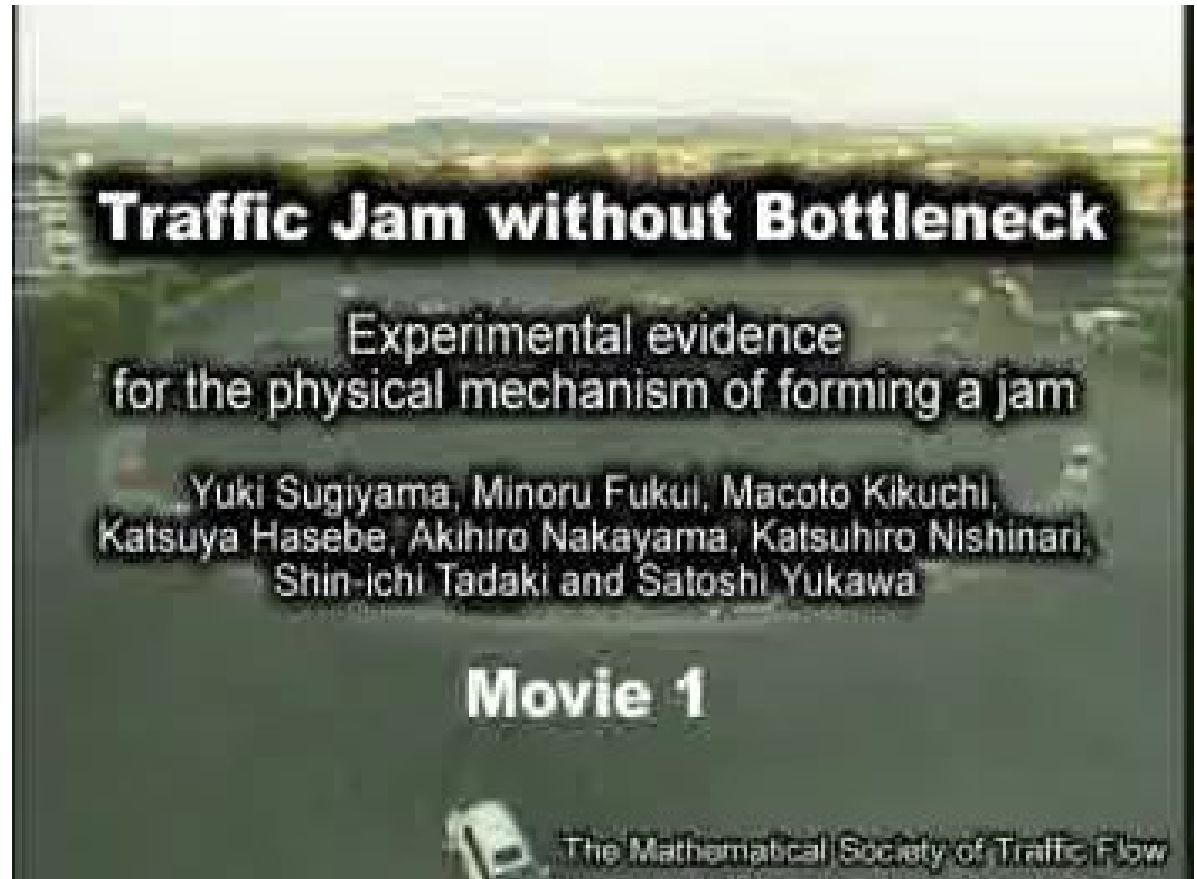
22 human drivers

## Instructions:

drive at 30 km/h  
/following its  
preceding vehicle

## Environment

Single lane  
No traffic lights,  
No stop signs,  
No lane changes.



# Benchmark Ring Road Experiment

## Traffic jams



## Setting:

21 human drivers

+ 1 AV

## Instructions:

drive at 30km/h  
/following its  
preceding vehicle

## Environment

Single lane  
No traffic lights,  
No stop signs,  
No lane changes.

Dissipation of stop-and-go traffic waves via control of a single autonomous vehicle



# Recent advances

## Traffic jams



## Reinforcement learning:

Wu, Cathy, et al., 2018 (MIT & Berkeley); Vinitzky, E., Kreidieh, A., Le Flem, L., Kheterpal, N., Jang, K., Wu, C., ... & Bayen, 2018, In Conference on robot learning.

## Adaptive and PDE control:

Yu, Huan, and Miroslav Krstic. Automatica, 2019. Yu, Huan, Saurabh Amin, and Miroslav Krstic. 2020, IEEE CDC.

## Hinf control:

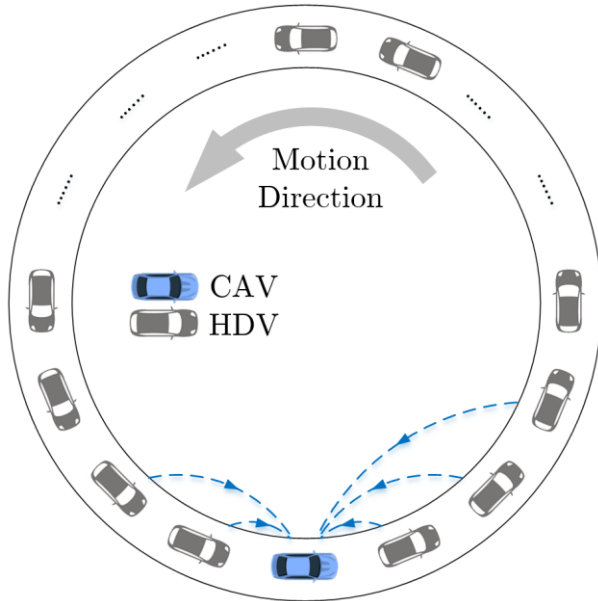
Mousavi, Shima Sadat, Somayeh Bahrami, and Anastasios Kouvelas. 2021 (ETH Zurich)



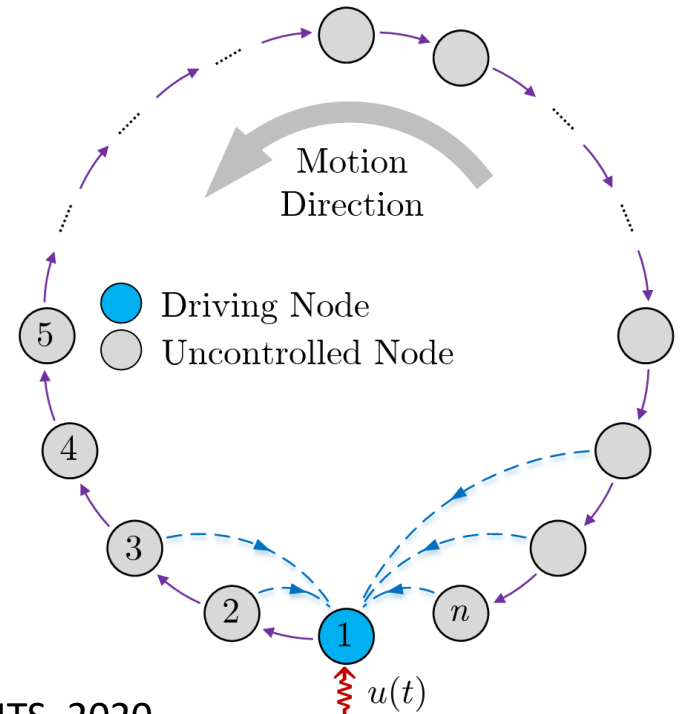
# Theoretical Evidence in mixed traffic

## □ Theoretical Evidence & Controller design

- Why does it work?
- Does it work in other setups (e.g., different number of HDVs, different human-driver behavior, open straight road scenario)?



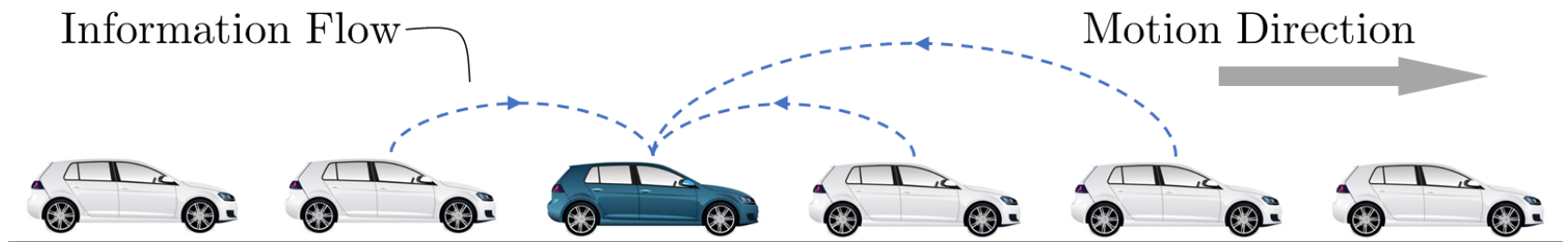
**Sparse network control**



# Scalable Control & Optimization

## □ Theoretical Evidence & Controller design

- How to design distributed controllers with limited communication?
- How to scale up the computation efficiency?



- Furieri, L., **Zheng**, Y., Papachristodoulou, A., & Kamgarpour, M. (2020). Sparsity invariance for convex design of distributed controllers. *IEEE Transactions on Control of Network Systems*. (**Best Student Paper Finalist**, ECC 2019)
- **Zheng**, Y., Fantuzzi, G., Papachristodoulou, A., Goulart, P., & Wynn, A. (2020). Chordal decomposition in operator-splitting methods for sparse semidefinite programs. *Mathematical Programming*, 180(1), 489-532.

# Today's talk

## Integrating Autonomy into Traffic Systems

### Part 1: Theoretic potential of autonomy in traffic

- Stabilizability of mixed traffic flow;
- Autonomous vehicles as mobile actuators in traffic networks;
- Leading Cruise Control (LCC)

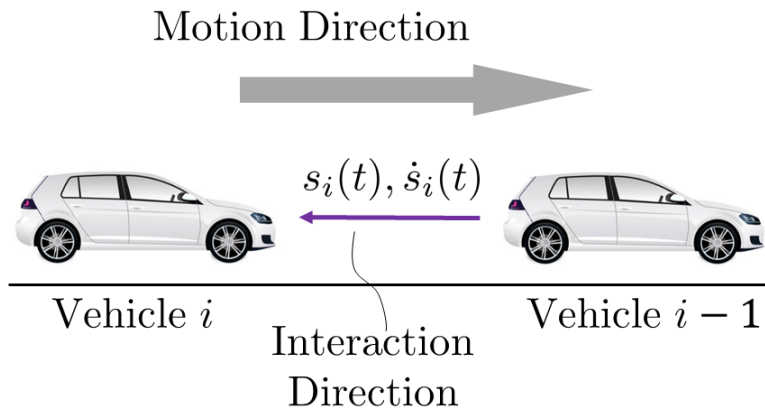
### Part 2: Practical design via control & optimization

- Convex design of distributed control over traffic network;
- Scalable optimization for large-scale convex problems;

# Mixed-autonomy in a ring road

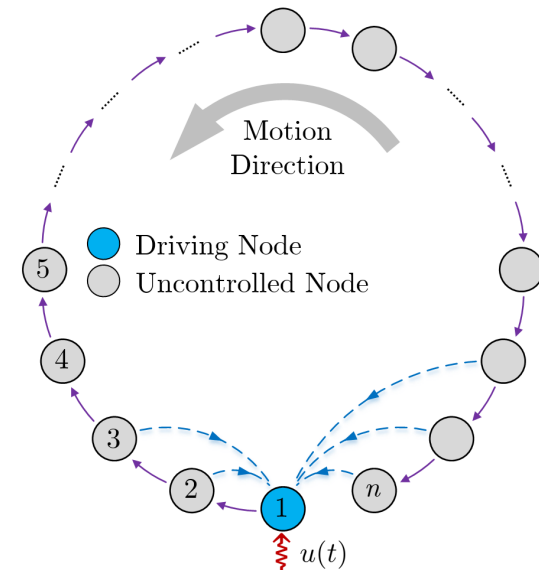
## □ System modeling

### 1. Human drivers → car-following dynamics

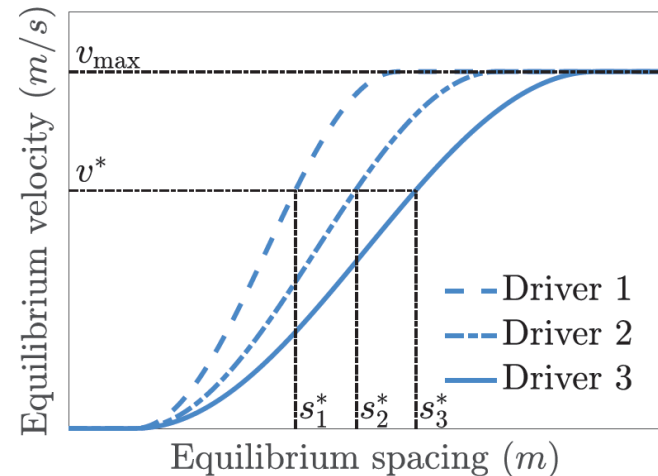


$$\dot{v}_i(t) = F_i(s_i(t), \dot{s}_i(t), v_i(t))$$

- $v_i(t)$ : Velocity of vehicle  $i$
- $s_i(t)$ : Spacing between vehicle  $i$  and vehicle  $i - 1$



$$0 = F_i(s_i(t), 0, v_i(t))$$



**Large spacing ↔ Large velocity**

# Mixed-autonomy in a ring road

## □ System modeling

### 2. Autonomous vehicle → direct control

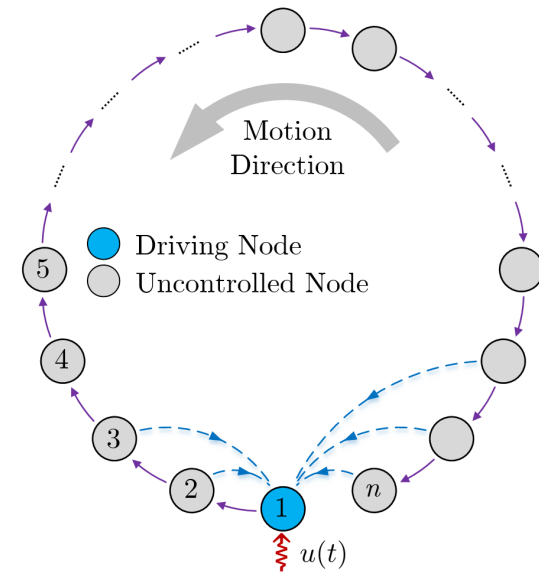
$$\begin{cases} \dot{s}_1(t) &= v_n(t) - v_1(t) \\ \dot{v}_1(t) &= u_1(t) \end{cases}$$

### 3. Assuming an equilibrium traffic state $v^*(t)$

$$\dot{x}(t) = Ax(t) + Bu(t),$$

where the system matrices have the following structure

$$A = \begin{bmatrix} C_1 & 0 & \dots & \dots & 0 & C_2 \\ A_2 & A_1 & 0 & \dots & \dots & 0 \\ 0 & A_2 & A_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & A_2 & A_1 & 0 \\ 0 & \dots & \dots & 0 & A_2 & A_1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$



A network system  
with only one  
controllable node

# Mixed-autonomy in a ring road

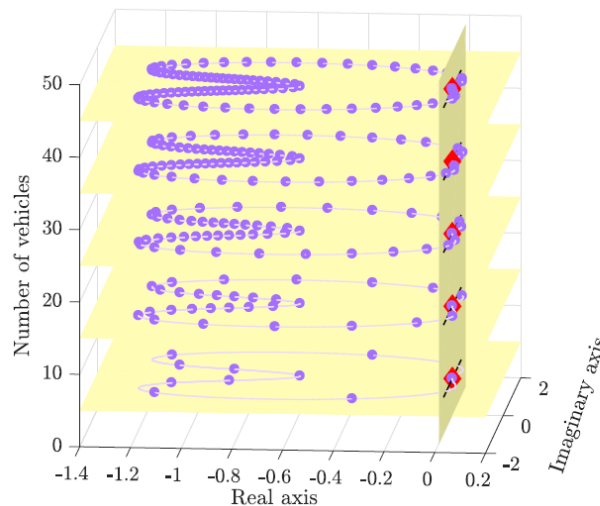
## □ Theoretical evidence 1: Unstable behavior

$$\dot{x}(t) = \hat{A}x(t)$$

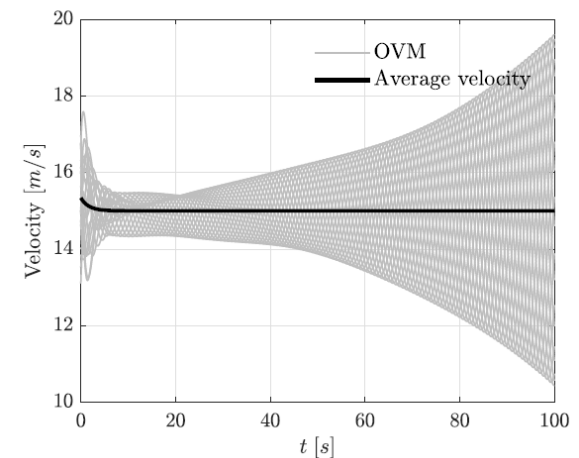
(Informal) The traffic system in a ring-road can be unstable if drivers' sensitivity to speed and spacing errors is small (e.g. Cui et al., 2017)

$$\alpha + 2\beta < \text{Constant}$$

Sensitivity to speed and spacing errors



Slow response to spacing; To catch up, it drives to a large velocity → **Oscillation**



# Mixed-autonomy in a ring road

## □ Theoretical evidence 2: Fundamental change of dynamics

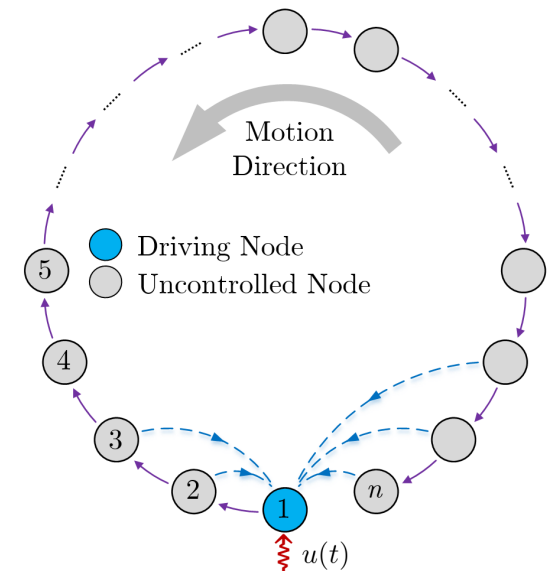
$$\dot{x}(t) = \hat{A}x(t)$$



**Theorem (zheng *et al.*, 2019): The mixed traffic system in the ring-road setup is not controllable, but stabilizable.**

1. Independent of the number of human-driven vehicles
2. Independent of car-following dynamics
3. Offer a strong control-theoretic support for the potential of autonomy in mixed traffic

Integrating autonomy is a fundamental change of traffic dynamics (more control freedom)!

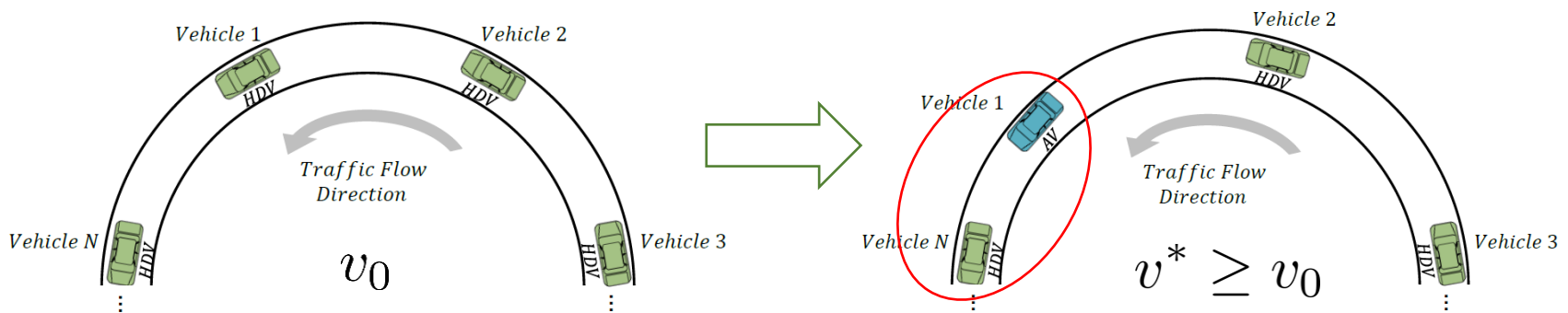


# Mixed-autonomy in a ring road

## □ Theoretical evidence 3: Beyond stabilization/increase traffic speed

**Theorem** (zheng *et al.*, 2019): **The global traffic velocity can be increased to a larger value:**

$$0 \leq v^* < v_{\max}$$



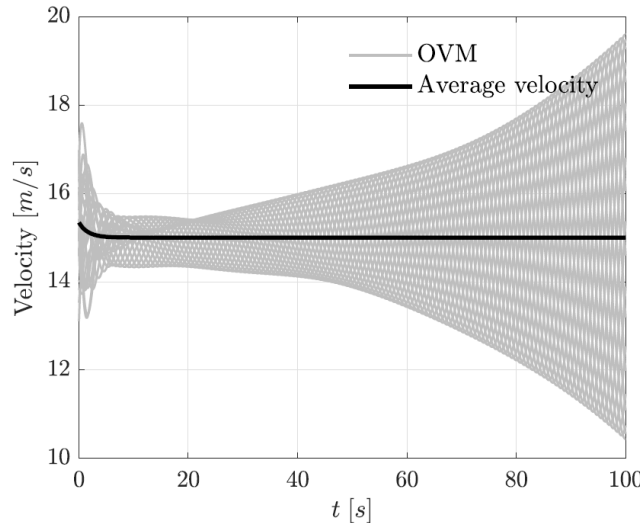
## □ Physical interpretation

- ✓ The AV can **change its own** spacing to **influence** other HDVs' spacing, and thus change traffic velocity  $v^*$ .



# Numerical Experiments with Nonlinear Dynamics

**Unstable** traffic system

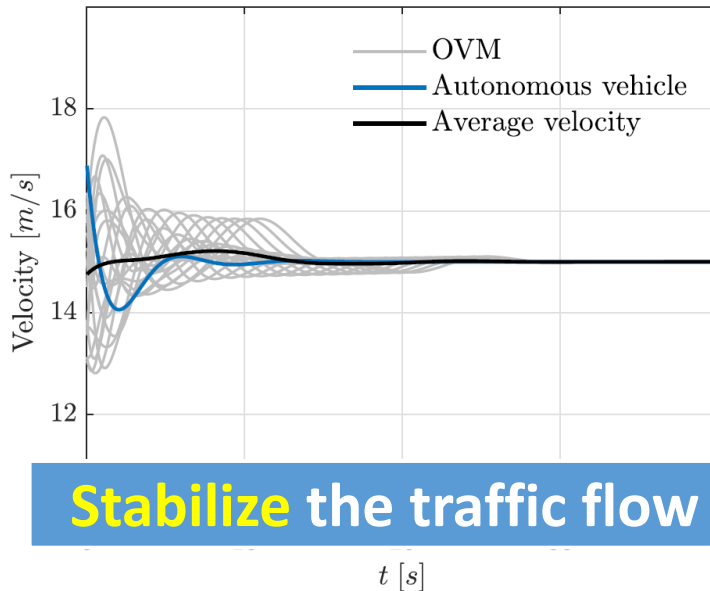


OVM: Optimal Velocity Model

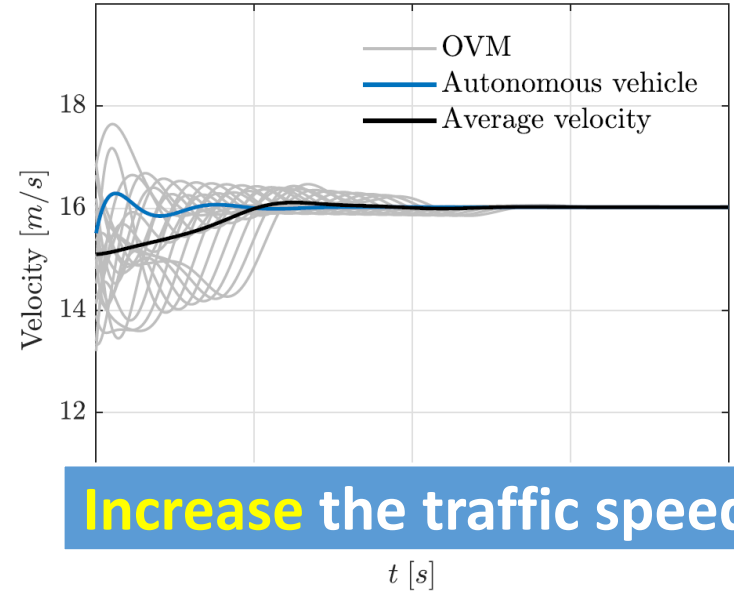
$$F_i = \alpha(V(s_i(t)) - v_i(t)) + \beta\dot{s}_i(t)$$

$$V(s) = \begin{cases} 0, & s \leq s_{st}, \\ f_v(s), & s_{st} < s < s_{go}, \\ v_{max}, & s \geq s_{go}, \end{cases}$$

$$f_v(s) = \frac{v_{max}}{2} \left( 1 - \cos\left(\pi \frac{s - s_{st}}{s_{go} - s_{st}}\right) \right).$$



**Stabilize** the traffic flow



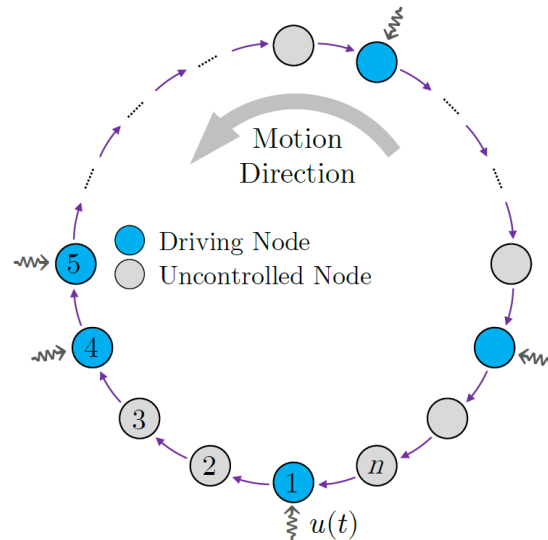
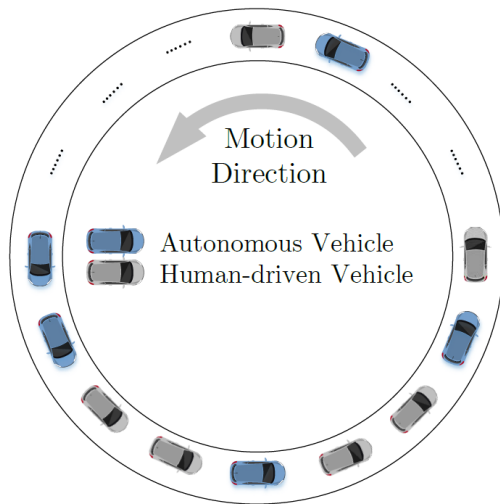
**Increase** the traffic speed

The existence of **5% AVs** (1 out of 20) can bring **6% improvement** on traffic velocity 17

# Integrating Autonomy: Multiple AVs



Main question: How to coordinate multiple autonomous vehicles in traffic flow? Is platooning the optimal choice?



Set function optimization

$$\max_S J(S)$$

$$S \subseteq \Omega, |S| = k$$

$\Omega = \{1, 2, \dots, n\}$ : all the vehicles

$S = \{i_1, \dots, i_k\} \subseteq \Omega$ :  $k$  autonomous vehicles

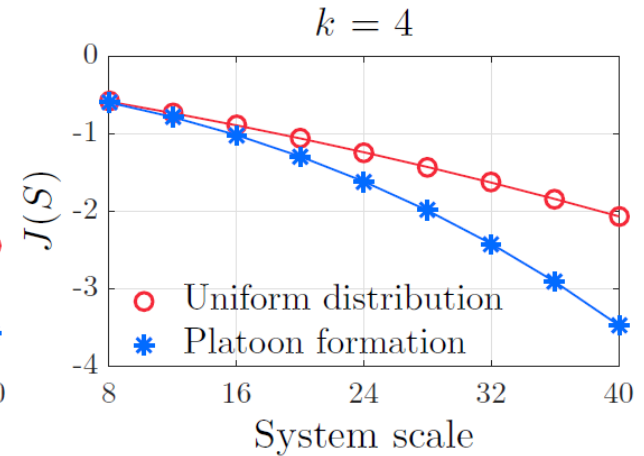
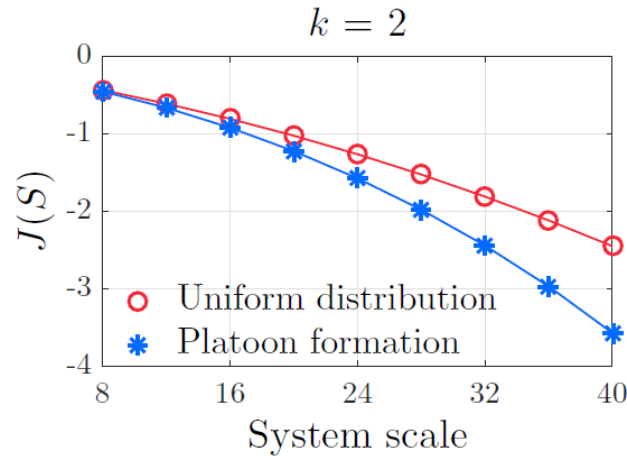
# Integrating Autonomy: Multiple AVs

Set function optimization

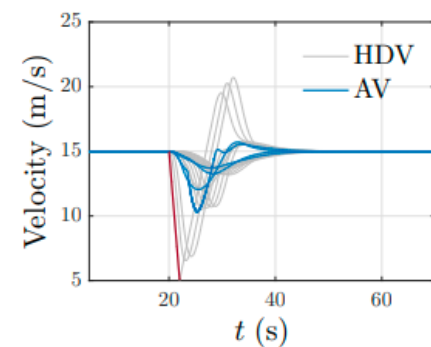
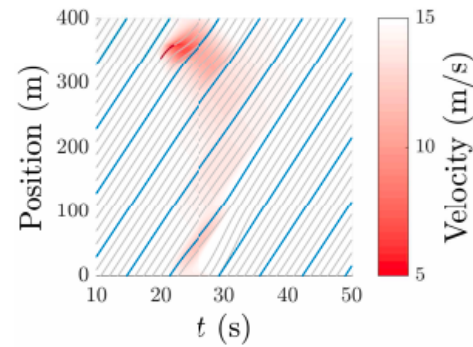
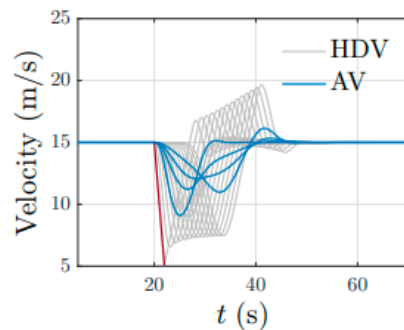
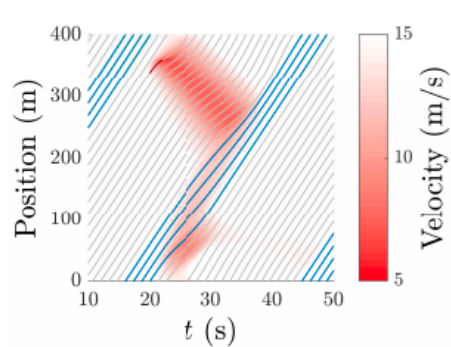
$$\max_S J(S)$$

$$S \subseteq \Omega, |S| = k$$

Platooning is not always optimal



## Simulation with Nonlinear Car-following Dynamics



Platoon formation:

$$S = \{9, 10, 11, 12\}$$

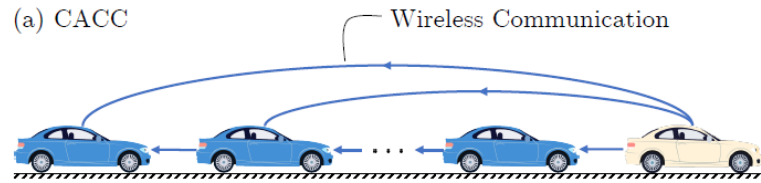
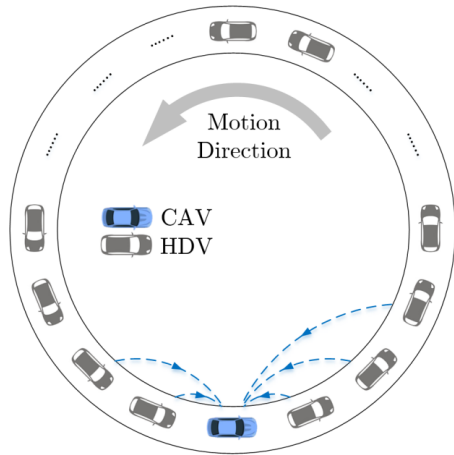
Li, Wang, & Zheng, (2020), IEEE TITS, under review

Uniform distribution:

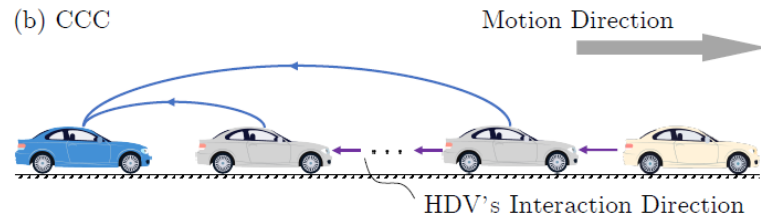
$$S = \{3, 8, 13, 18\}$$

# Integrating Autonomy in Open-straight roads

## Closed-ring road setup

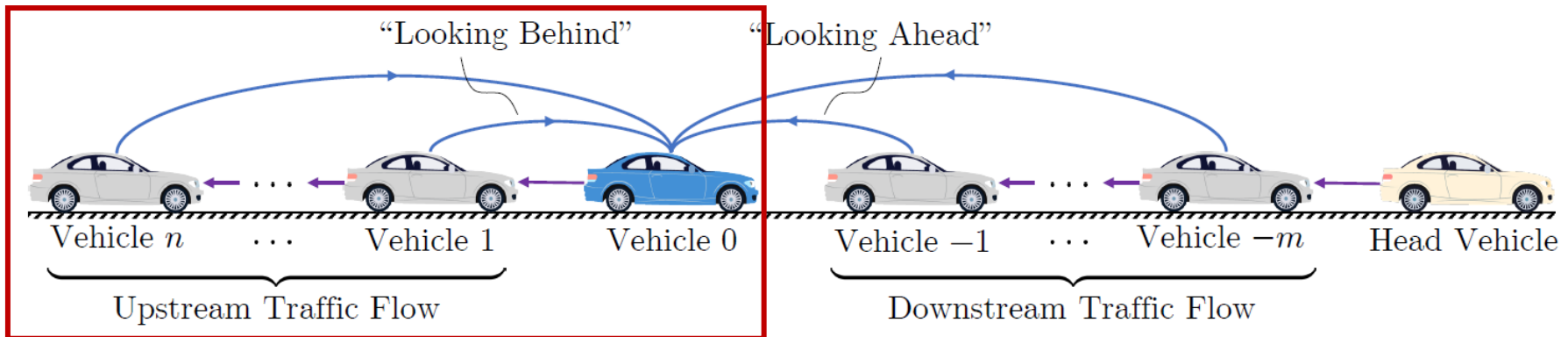


## CACC: Fully-autonomous scenario



## Connected Cruise Control: downstream traffic flow

## ➤ Leading Cruise Control



Lead the motion of the vehicles behind

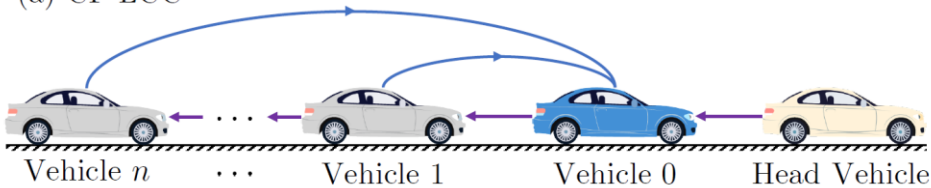


Adapt to the motion of the vehicles ahead

# Leading Cruise Control (LCC)

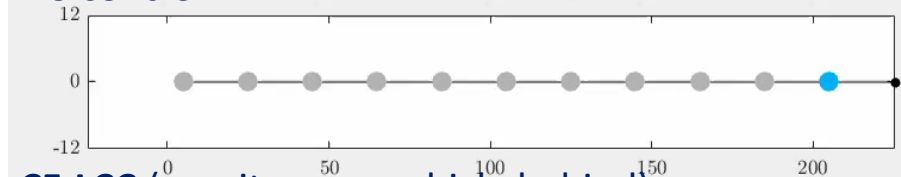
## Special case 1: car-following LCC

(a) CF-LCC

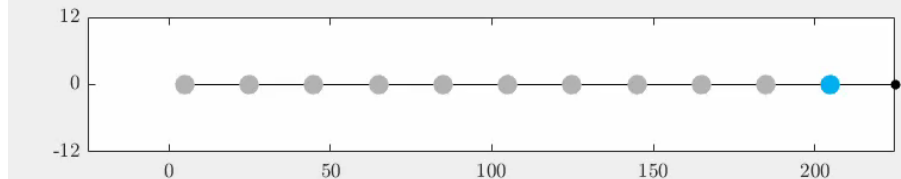


No control

Time = 15.0 s



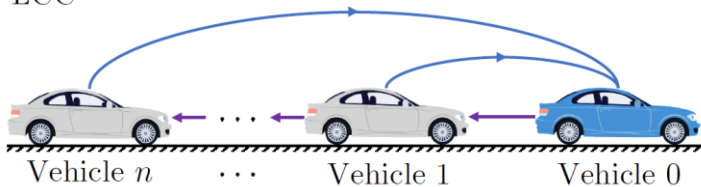
CF-LCC (monitor one vehicle behind)



Reduce velocity perturbations by **28%**

## Special case 2: free driving LCC

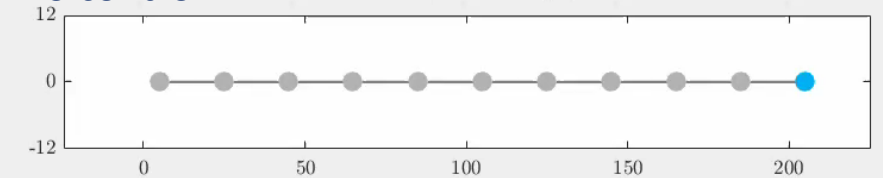
(b) FD-LCC



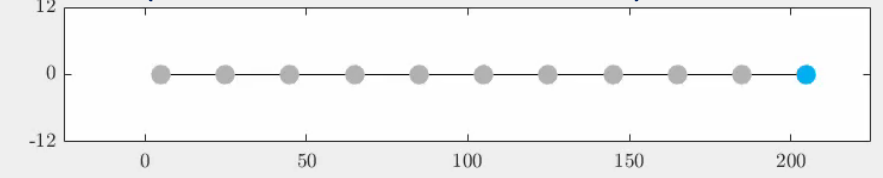
1. The motion after AV is controllable (leading motion behind)
2. String stability can be improved (attenuating perturbation ahead)

No control

Time = 15.0 s



FD-LCC (monitor one vehicle behind)



Reduce velocity perturbations by **35%**

# Today's talk

## Integrating Autonomy into Traffic Systems

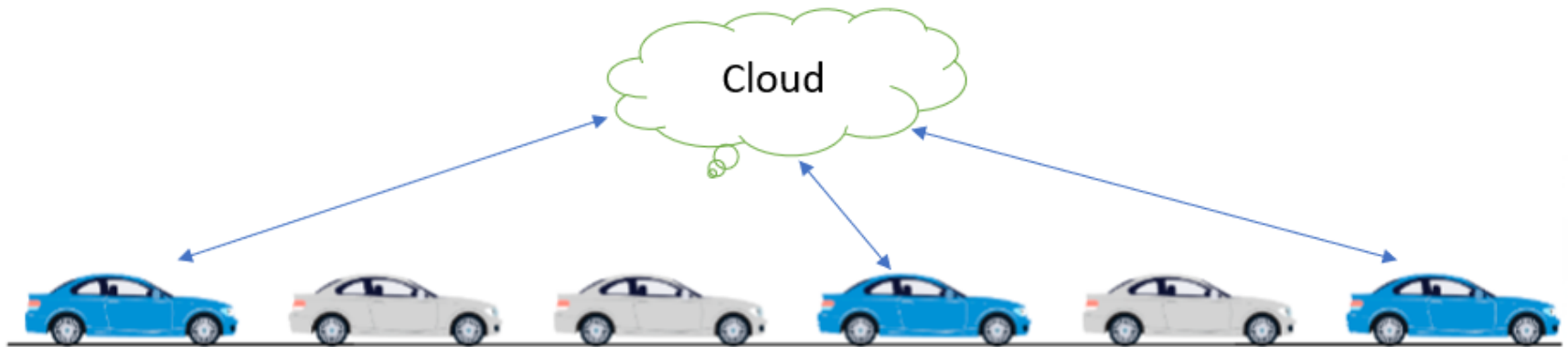
### Part 1: Theoretic potential of autonomy in traffic

- Stabilizability of mixed traffic flow;
- Autonomous vehicles as mobile actuators in traffic networks;
- Leading Cruise Control (LCC)

### Part 2: Practical design via control & optimization

- Convex design of distributed control over traffic network;
- Scalable optimization for large-scale convex problems;

# General Procedure



**Control Problem  
Formulation**



**Convex reformulation  
as LMI or SDP**



**Call a numerical  
solver**

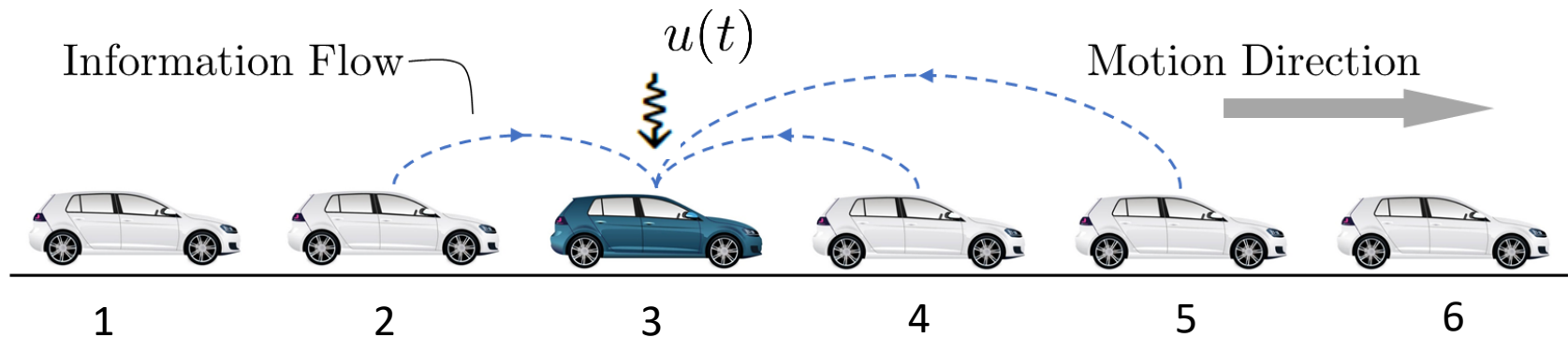
**Challenge 1: How to recover  
convexity**

**Challenge 2: How to deal large-  
scale problems (Scalability)**

# Problem formulation: distributed controller

## □ Why distributed?

- No need of a centralized coordinator
- Allow for local communication



$$u(t) = -(k_1 x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4 + k_5 x_5 + k_6 x_6)$$

$$k_1 = k_6 = 0$$

## Compact form

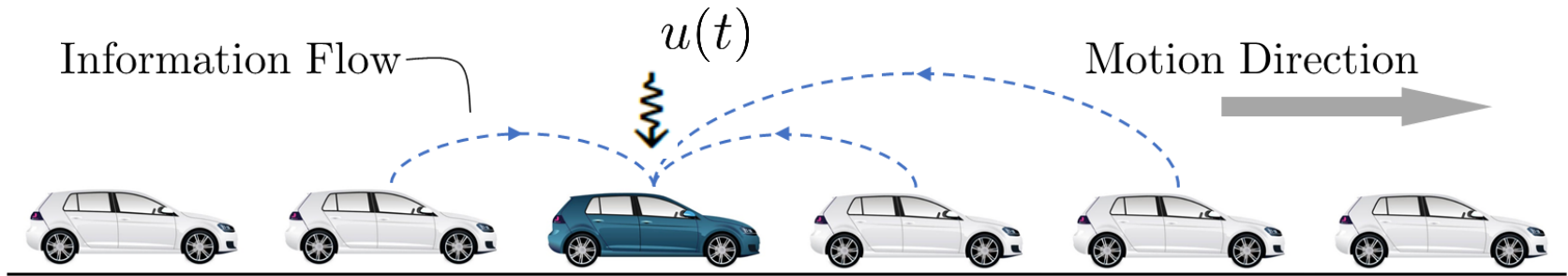
$$u(t) = -Kx(t), \quad K \in \text{Sparse}(S)$$

$$S = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

encodes local communication



# Problem formulation: distributed controller



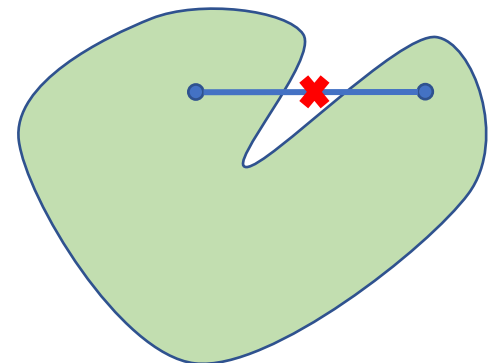
$$\begin{aligned} & \min_K J(K) && \longrightarrow \text{System performance} \\ & \text{subject to } K \in \mathcal{C}_{\text{stab}}, && \longrightarrow \text{Stable controller} \\ & K \in \text{Sparse}(S) && \longrightarrow \text{Distributed controller} \end{aligned}$$

- This is a **non-convex optimization problem**

$$\exists K_1 \in \mathcal{C}_{\text{stab}}, K_2 \in \mathcal{C}_{\text{stab}}$$

$$\Rightarrow \frac{1}{2}(K_1 + K_2) \notin \mathcal{C}_{\text{stab}}$$

- The presence of the sparsity constraint makes the problem **even more challenging** (NP-hard in general).



# Previous work on distributed control

## □ 90's: Feasibility & Stabilization

- 1) **Structural controllability:** Glover & Silverman, [TAC 1976](#); Wang & Davison, [TAC 1973](#); Davison, [Automatica 1977](#); Mayeda and Yamada, [SICON 1979](#), etc.
- 2) **Decentralized/distributed fixed mode:** Anderson & Clements, [TAC 1981](#); Sezer & Šiljak, [SCL 1981](#); Davison & Özgüner, [Automatica 1983](#); etc.
- 3) **Decentralized stabilization & pole placement:** Davison & Chang, [TAC 1995](#); Ravi et al, [TAC 1995](#)
- 4) **Early survey paper:** Sandell, Varaiya, Athans & Safonov, [TAC 1978](#).

## □ Late 90's- Now: Performance enhancement via optimization

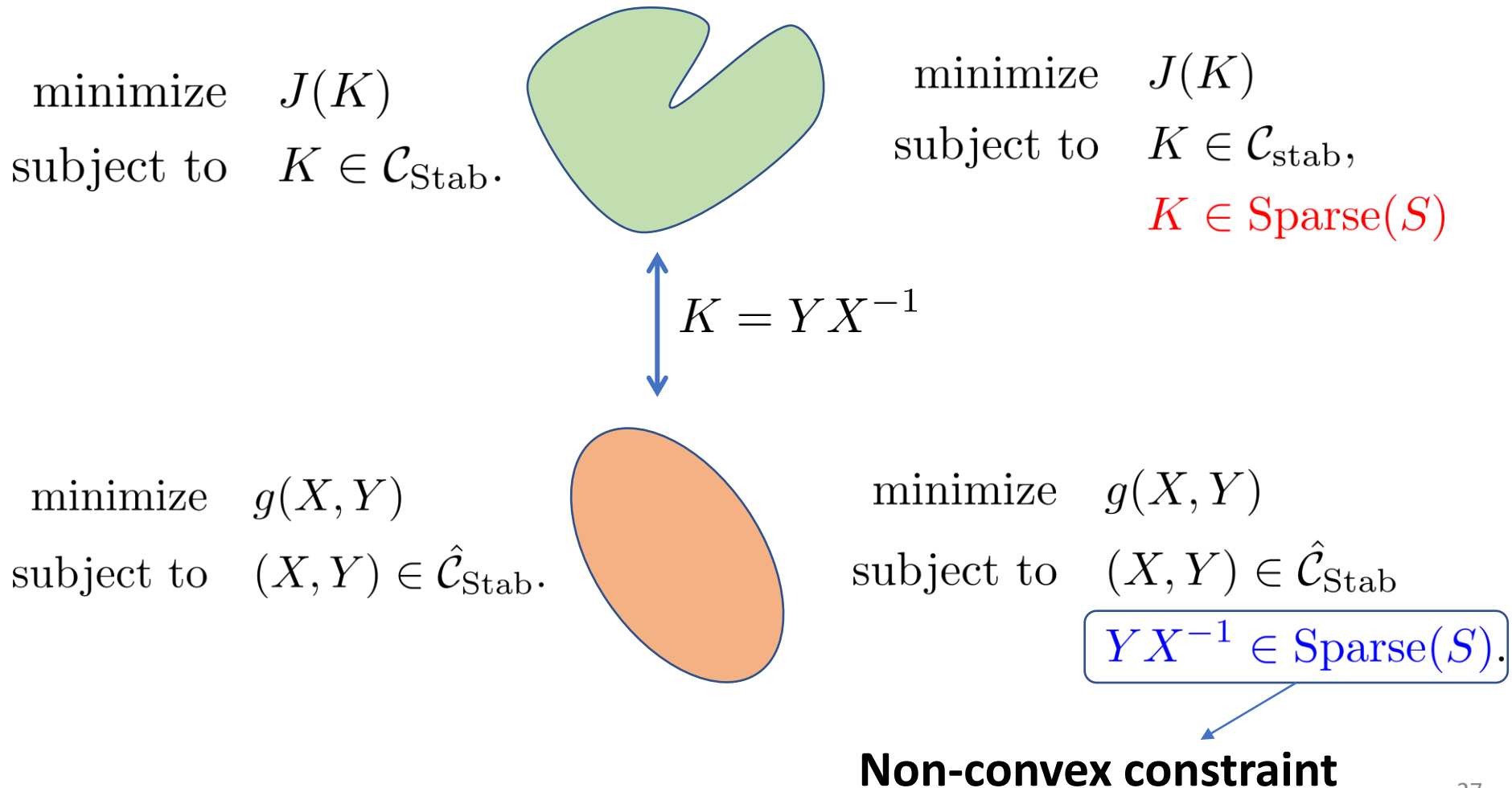
- 1) **Exact solutions for special classes of systems:** Quadratic Invariance (Rotkowitz & Lall, [TAC 2005](#)); Partially ordered sets (Shah & Parrilo, [TAC 2013](#));
- 2) **Tractable convex approximation:** Dvijotham et al, [TCNS 2015](#); Fazelnia et al, [TAC 2016](#);
- 3) **Suboptimal solutions using iterative algorithms:** Fu, Fardad, & Jovanovic, [TAC 2011](#);
- 4) **Structure regularization and system-level synthesis:** Jovanović & Dhingra, [2016](#); Wang et al., [TAC 2019](#);

**Recover**  
**Convexity**

A new framework based on Sparsity Invariance  
for convex design of distributed control

# Change of Variables

- Do not optimize the controller  $K$  directly:** Convex reformation via a change of variables (convex SDP; Boyd *et al.*, 1994);



# Sparsity Invariance

$$\begin{array}{ll} \text{minimize} & g(X, Y) \\ \text{subject to} & (X, Y) \in \hat{\mathcal{C}}_{\text{Stab}} \end{array}$$

$$YX^{-1} \in \text{Sparse}(S).$$

→ **Non-convex constraint**

## Sparsity invariance (SI)

$$X \in \text{Sparse}(R), Y \in \text{Sparse}(T)$$

⇒

$$K = YX^{-1} \in \text{Sparse}(S)$$

## Convex approximation

$$\text{minimize } g(X, Y)$$

$$\text{subject to } (X, Y) \in \hat{\mathcal{C}}_{\text{Stab}}$$

$$X \in \text{Sparse}(R)$$

$$Y \in \text{Sparse}(T).$$

- **Translate the constraint on the controller to separate constraints on new decision variables**

**Recover**  
**Convexity**

# Sparsity Invariance

## Sparsity invariance (SI)

$$X \in \text{Sparse}(R), Y \in \text{Sparse}(T)$$

$\Rightarrow$

$$K = YX^{-1} \in \text{Sparse}(S)$$

## Special case: the widely used diagonal assumption

$R = I, T = S$  is a trivial choice; (Geromel et al., 1994; Conte et al., 2012; Rubio et al., 2013;)

1. A full characterization

$$TR^{n-1} \leq S$$

2. A practical optimal design of the patterns  $R, T$

**Further  
contributions**

# Unified framework for distributed control

$$\begin{aligned} X \in \text{Sparse}(R), Y \in \text{Sparse}(T) \\ \text{Sparsity invariance (SI)} \quad \Rightarrow \\ K = YX^{-1} \in \text{Sparse}(S) \end{aligned}$$

**Recover**  
**Convexity**

A new framework based on Sparsity Invariance  
for convex design of distributed control

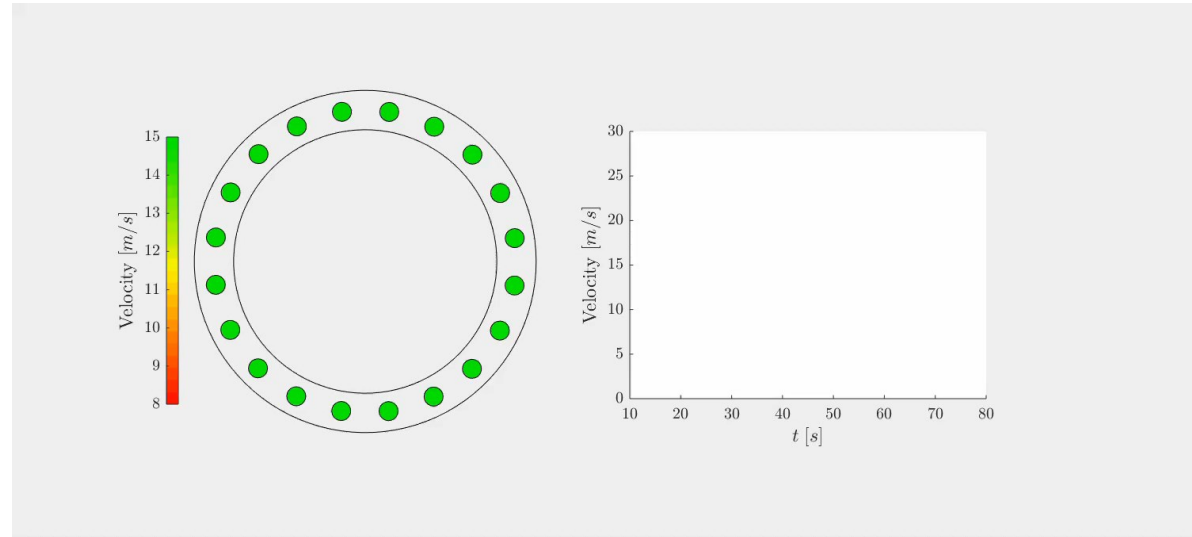
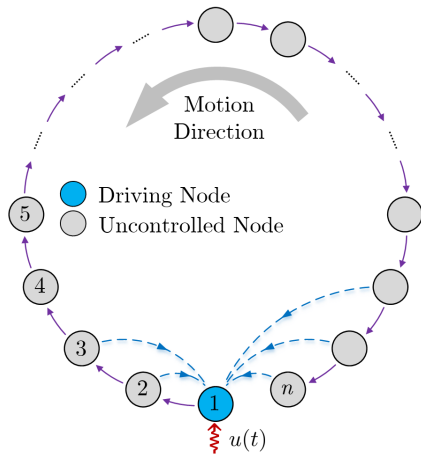
## Static feedback

- Strictly better than the widely used **diagonal approximation strategy** (Geromel et al., 1994; Conte et al., 2012; Rubio et al., 2013; Han et al., 2017)

## Dynamical feedback (past information + memory)

- Guaranteed to be optimal when a notion of **Quadratic Invariance (QI)** holds (Rotkowitz & Martins, 2012)
- Best known performance for non-QI cases

# Numerical Experiments with Nonlinear Dynamics

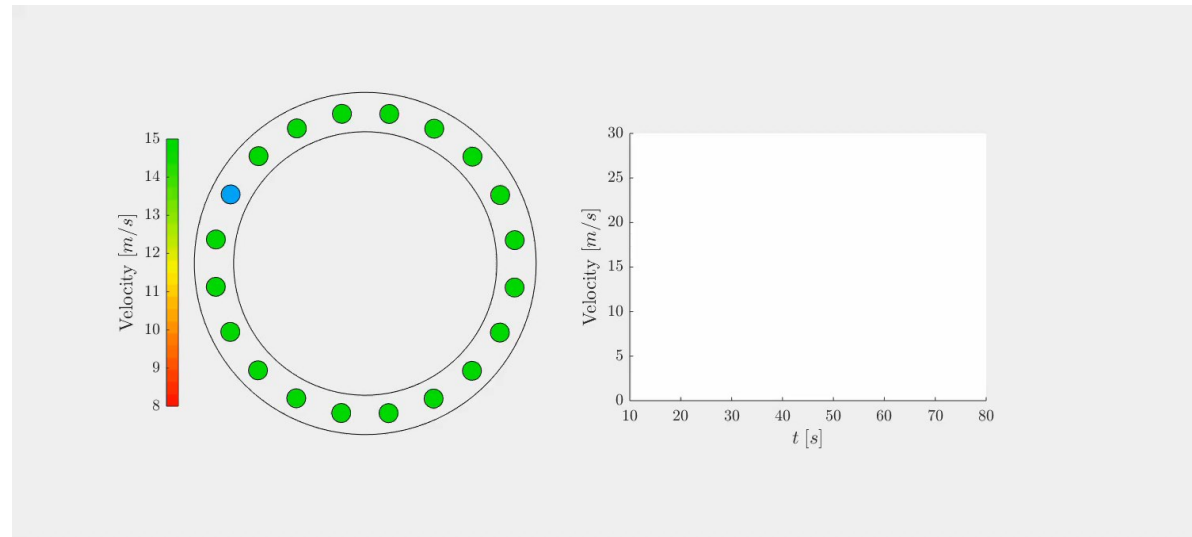


## OVM: Optimal Velocity Model

$$F_i = \alpha(V(s_i(t)) - v_i(t)) + \beta \dot{s}_i(t)$$

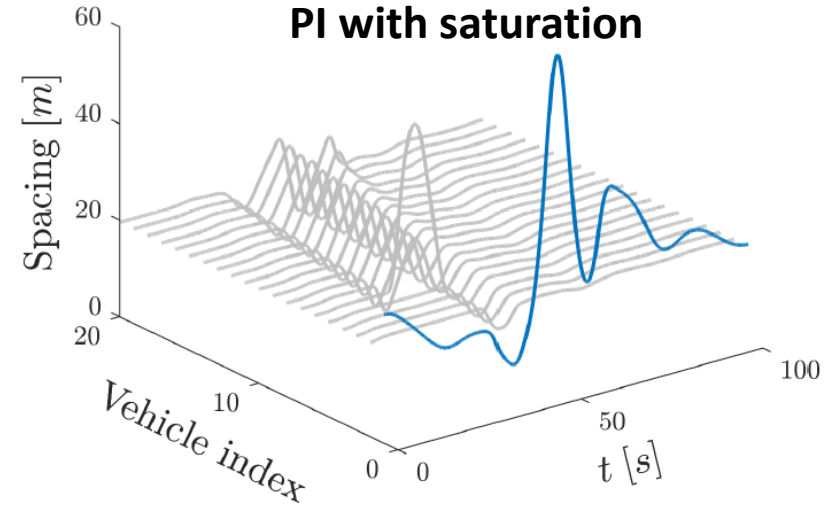
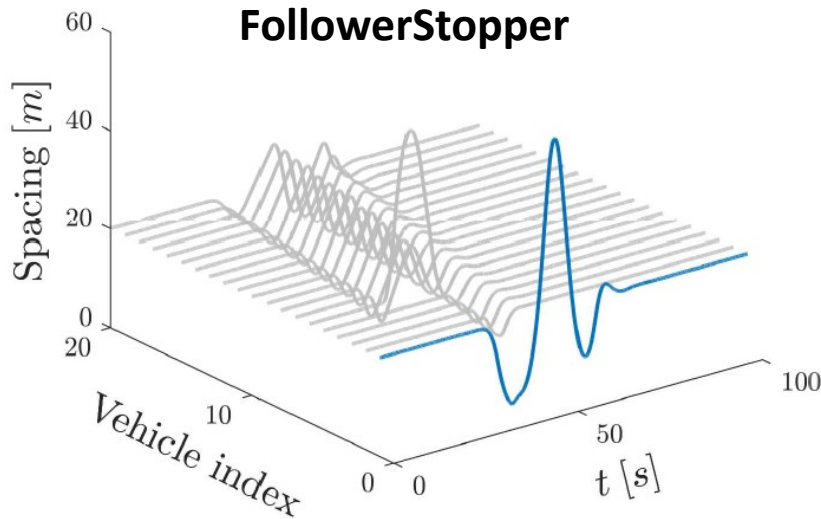
$$V(s) = \begin{cases} 0, & s \leq s_{st}, \\ f_v(s), & s_{st} < s < s_{go}, \\ v_{max}, & s \geq s_{go}, \end{cases}$$

$$f_v(s) = \frac{v_{max}}{2} \left( 1 - \cos\left(\pi \frac{s - s_{st}}{s_{go} - s_{st}}\right) \right).$$

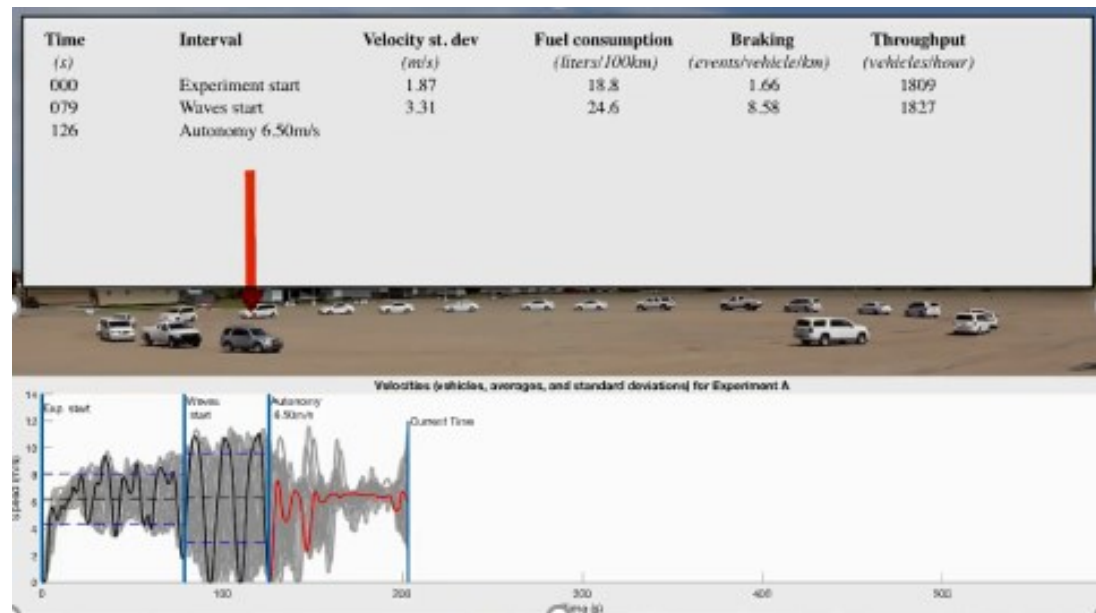


# Comparison with existing methods

## Comparison with the heuristic methods in Stern et al. 2018



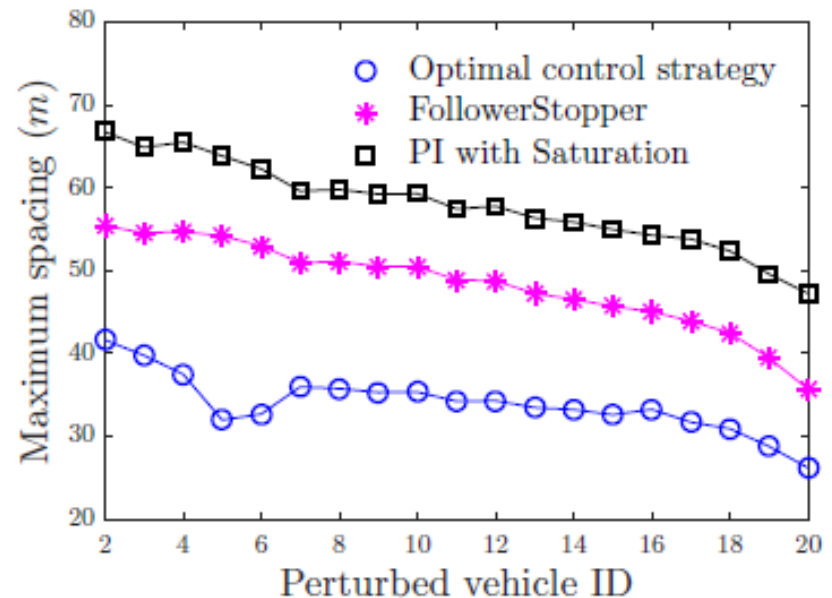
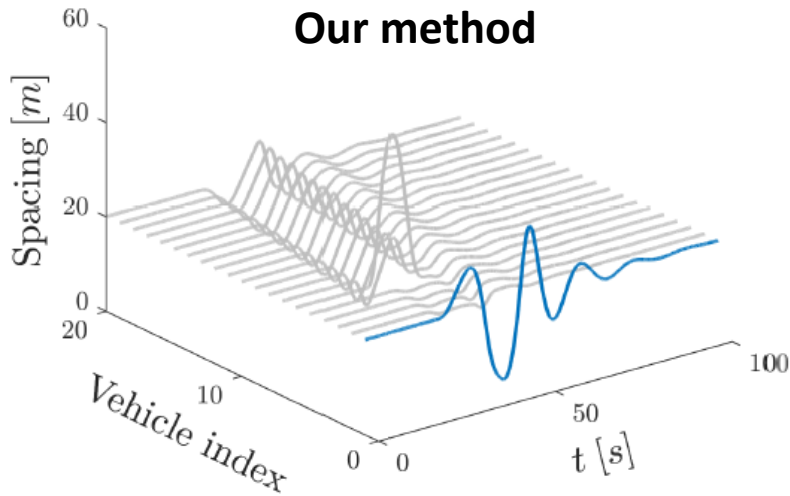
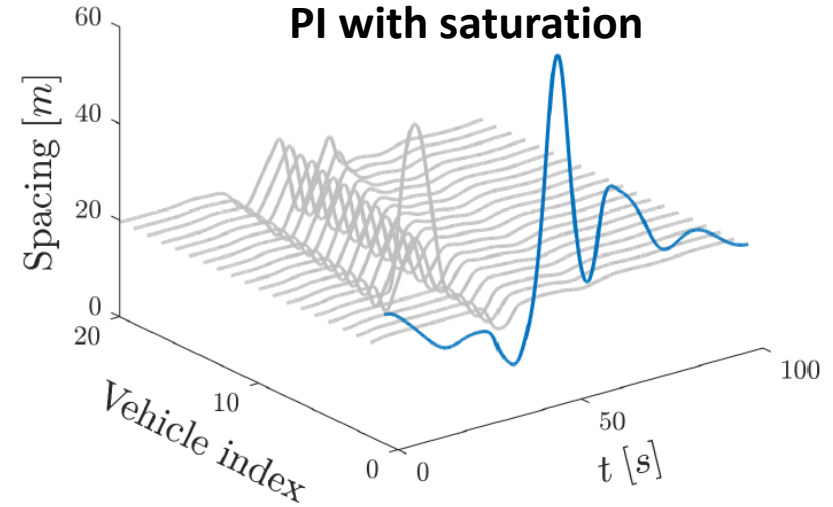
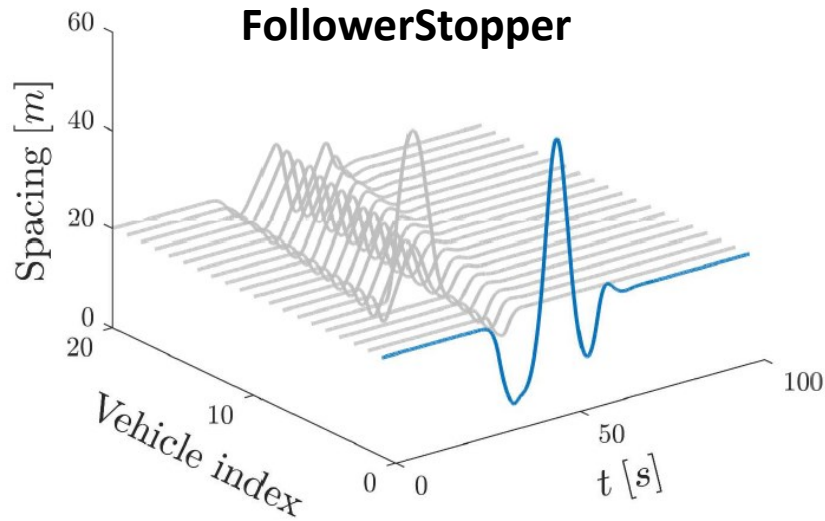
- These methods are conservative
- They lead to large spacing, which may cause other vehicle to cut-in



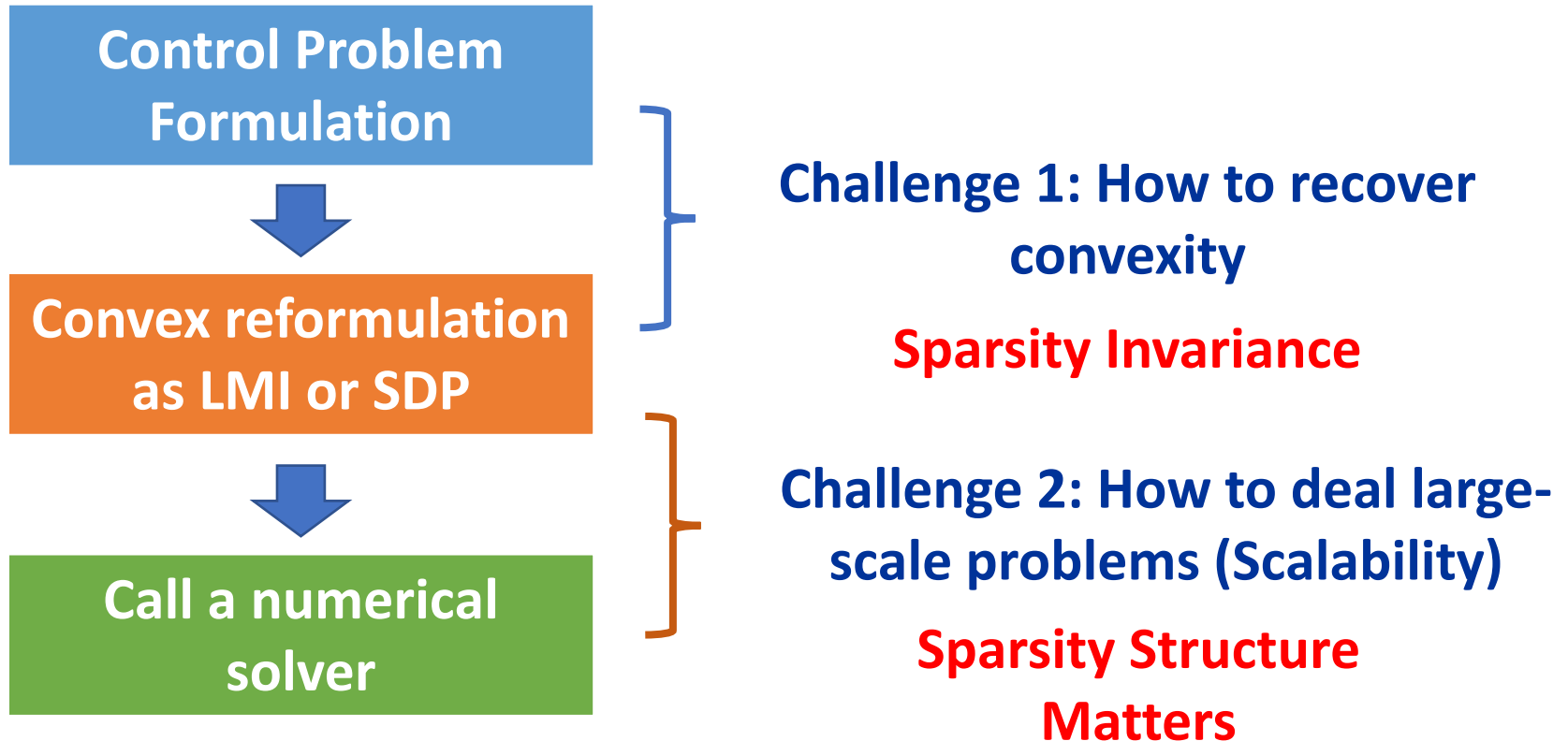


# Comparison with existing methods

## Comparison with the heuristic methods in Stern et al. 2018



# General Procedure



# General convex optimization

**Semidefinite program**

$$\begin{aligned} & \max_{y, Z} \quad \langle b, y \rangle \\ & \text{subject to} \quad Z + \sum_{i=1}^m A_i y_i = C, \\ & \quad \quad \quad Z \in \mathbb{S}_+^n \end{aligned}$$

- **Applications:** control theory, fluid mechanics, polynomial optimization, combinatorics, operations research, finance.
- **Standard interior-point solvers:** SeDuMi, SDPT3, Mosek (suitable for small and medium-sized problems; say  $n < 1000$ );
- **Practical large-scale instances:** Standard interior point solvers will fail on large-scale problems (say  $n$  being a few thousands).

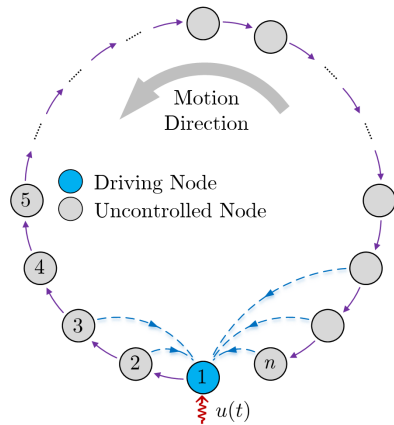
**Sparsity Structure Matters**

Decompose a big positive semidefinite constraint into multiple smaller ones

# Sparsity Structure

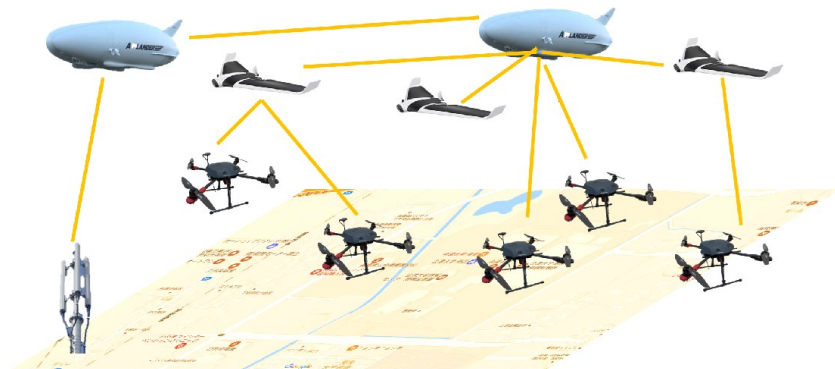
Sparsity structure appears in many places of real cyber-physical systems

## System dynamics data

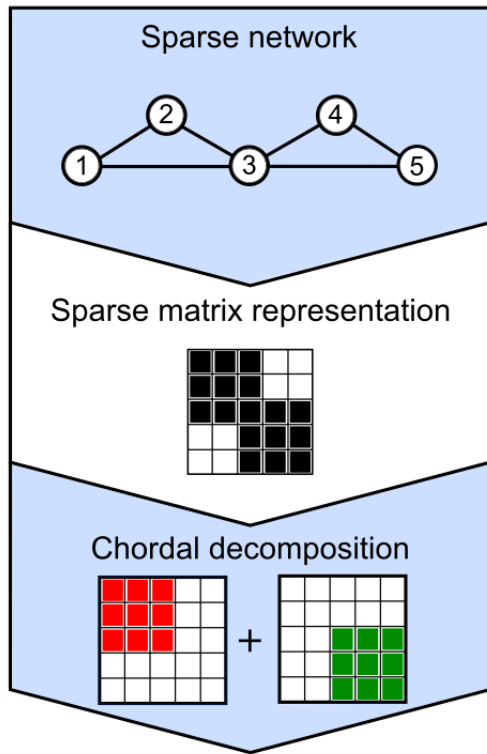


$$A = \begin{bmatrix} C_1 & 0 & \dots & \dots & 0 & C_2 \\ A_2 & A_1 & 0 & \dots & \dots & 0 \\ 0 & A_2 & A_1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & A_2 & A_1 & 0 \\ 0 & \dots & \dots & 0 & A_2 & A_1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

## Sparse communication

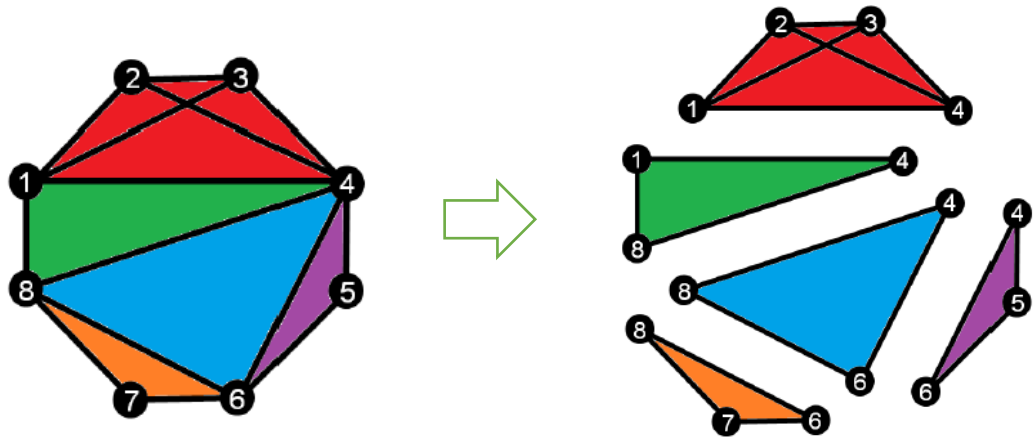


# Graph Decomposition



$$A = \underbrace{\begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}}_{\succeq 0} = \underbrace{\begin{bmatrix} 3 & 1 & 0 \\ 1 & 0.5 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\succeq 0} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 1 \\ 0 & 1 & 3 \end{bmatrix}}_{\succeq 0}$$

## Chordal graph decomposition



Vandenbergh & Andersen (2015).

- This allows for the decomposition of a big positive semidefinite constraint
- Exploiting this decomposition  $\rightarrow$  a new scalable algorithm for sparse SDP (Zheng, Y., et al. *Math. Prog.*, 2019)

# Open-source Solver: CDCS

## CDCS: Cone Decomposition Conic Solver

- Open-source MATLAB solver for sparse conic optimization problems (SDPs, QPs, LPs, SOCPs, etc)
- Can be called from modeling packages, like YALMIP and SOSTOOLS.
- Available from: <https://github.com/oxfordcontrol/CDCS>

## Numerical comparison

1. Standard interior-point solver: **SeDuMi**
2. State-of-the-art first-order solver: **SCS**

# Open-source Solver: CDCS

## Case 1: Test on sparse benchmark problems (from Andersen, et al, 2010)

	rs1555			rs1907		
	Time (s)	# Iter.	Objective	Time (s)	# Iter.	Objective
SeDuMi (high)	***	***	***	***	***	***
SeDuMi (low)	***	***	***	***	***	***
SCS (direct)	139,314	2000	34.20	50,047	2000	45.89
CDCS-primal	1721	2000	61.22	330	349	62.87
CDCS-dual	317	317	69.54	271	252	63.30
CDCS-hsde	1413	2000	61.36	393	414	63.14

Entries marked \*\*\* indicate that the problem could not be solved due to memory limitations

Problem instance: rs1907

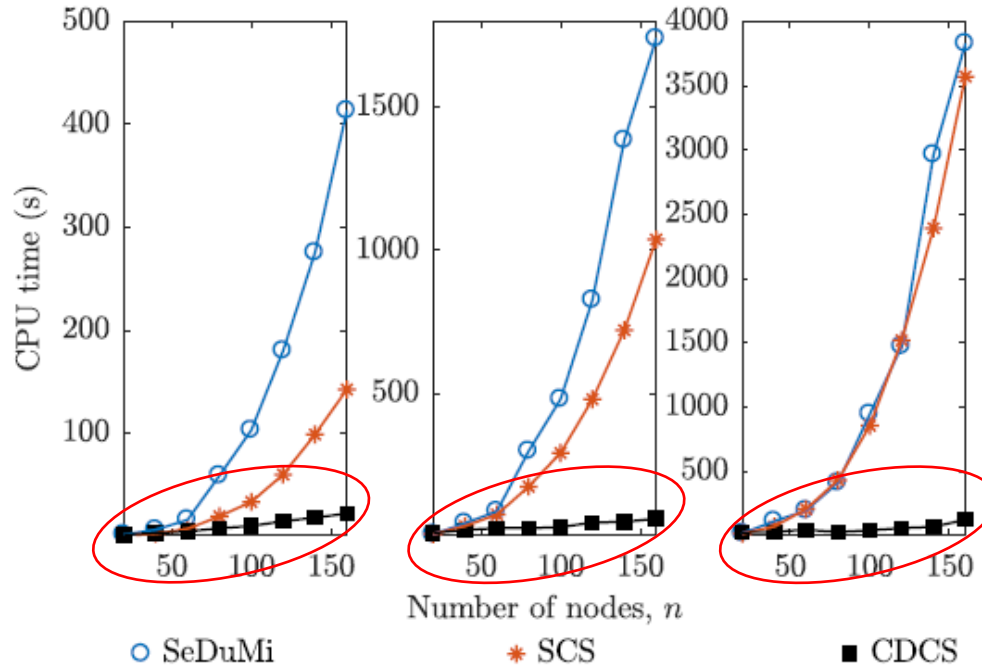
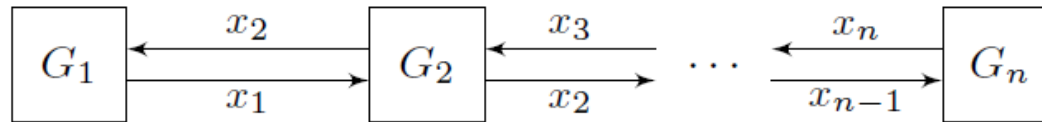
- PSD size  $5357 \times 5357$
- > **10 million** decision variables

- ✓ SeDuMi ran out of memory
- ✓ The first-order solver SCS took over **13 hours** to return a solution
- ✓ **CDCS** took **6 minutes** to get a solution; **100 × speedup!**

Exploiting sparsity achieves massive scalability!

# Open-source Solver: CDCS

## Case 2: Test on stability/H2/Hinf analysis of linear cascaded systems



Order of magnitude  
faster  
→ Massive Scalability



# Open-source Solver: CDCS

## Large-scale practical problems

oxfordcontrol / CDCS

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An open-source MATLAB® ADMM solver for partially decomposable conic optimization programs. [Edit](#)

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116 commits 5 branches 0 packages 1 release 4 contributors LGPL-3.0

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### □ Signal recovery problem

- Fosson, S. M., & Abuabiah, M. (2019). Recovery of binary sparse signals from compressed linear measurements via polynomial optimization. *IEEE Signal Processing Letters*.

### □ Optimal power flow problem

- Eltved, A., Dahl, J., & Andersen, M. S. (2018). On the robustness and scalability of semidefinite relaxation for optimal power flow problems. *Optimization and Engineering*, 1-18.

### □ Nonlinear systems analysis

- Driggs & Fawzi (2019). "AnySOS: An anytime algorithm for semidefinite programming" *IEEE CDC*, 1-6.

# Conclusion

# Two main takeaways

## Theoretic potential of autonomy in traffic

- Mixed traffic systems is always **stabilizable**;
- Autonomous vehicles can not only **smooth traffic wave**, but also guide traffic velocity to a **higher value**;
- Autonomous vehicles can change traffic dynamics fundamentally (**Leading Cruise Control**)

## Integrating Autonomy via Control and Optimization

- **Convexity** of distributed control: a new framework based on sparsity invariance
- **Scalability** of convex optimization: Sparsity-exploiting methods based on graph decomposition

# References

## Mixed autonomy analysis

1. **Zheng, Y.**, Wang, J., & Li, K. (2020). Smoothing traffic flow via control of autonomous vehicles. *IEEE Internet of Things Journal*, 7(5), 3882-3896.
2. Li, K., Wang, J., & **Zheng, Y.** (2020). Cooperative Formation of Autonomous Vehicles in Mixed Traffic Flow: Beyond Platooning. arXiv preprint arXiv:2009.04254.
3. Wang, J., **Zheng, Y.**, Xu, Q., Wang, J., & Li, K. (2020). Controllability Analysis and Optimal Control of Mixed Traffic Flow with Human-driven and Autonomous Vehicles. *IEEE Transactions on Intelligent Transportation Systems*, accepted.
4. Wang, J., Zheng, Y., Chen, C., Xu, Q., & Li, K. (2021). Leading cruise control in mixed traffic flow: System modeling, controllability, and string stability. *IEEE Transactions on Intelligent Transportation Systems*.

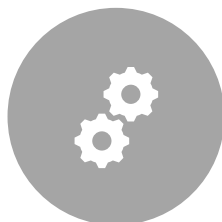
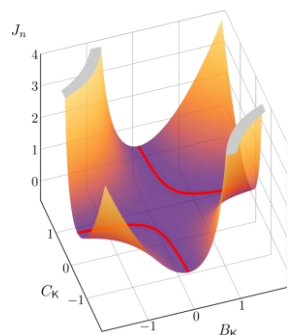
## Controller Design & Scalability

1. Furieri, L., **Zheng, Y.**, Papachristodoulou, A., & Kamgarpour, M. (2019). Sparsity Invariance for Convex Design of Distributed Controllers. *IEEE Transactions on Control of Network Systems*, accepted. 1-11. (**Best Student Paper Award Finalist**, conference version)
2. **Zheng, Y.**, Furieri, L., Papachristodoulou, A., Li, N., & Kamgarpour, M. (2020). On the Equivalence of Youla, System-Level, and Input–Output Parameterizations. *IEEE Transactions on Automatic Control*, 66(1), 413-420.
3. **Zheng, Y.**, Fantuzzi, G., Papachristodoulou, A., Goulart, P., & Wynn, A. (2020). Chordal decomposition in operator-splitting methods for sparse semidefinite programs. *Mathematical Programming*, 1-44.

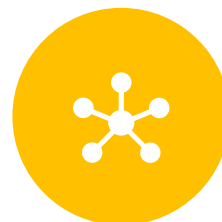
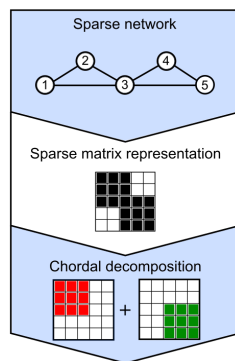
# Join the SOC lab at UC San Diego!



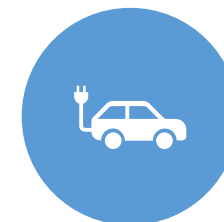
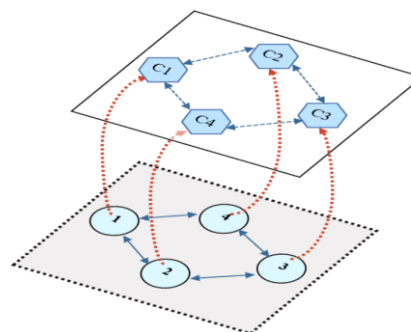
**Data-driven  
and learning-  
based control**



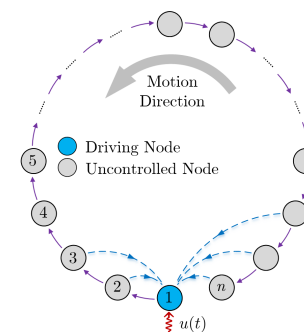
**Sparse conic  
optimization**



**Scalable  
distributed  
control**

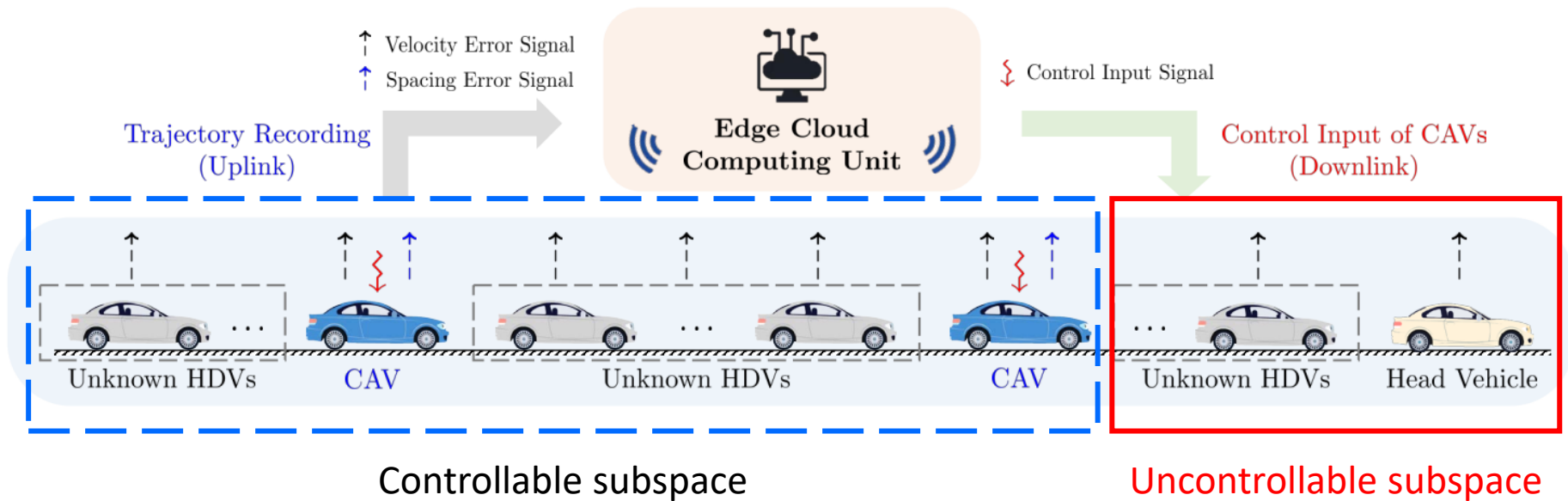


**Connected and  
autonomous  
vehicles (CAVs)**



Extra slides

# Data-driven MPC



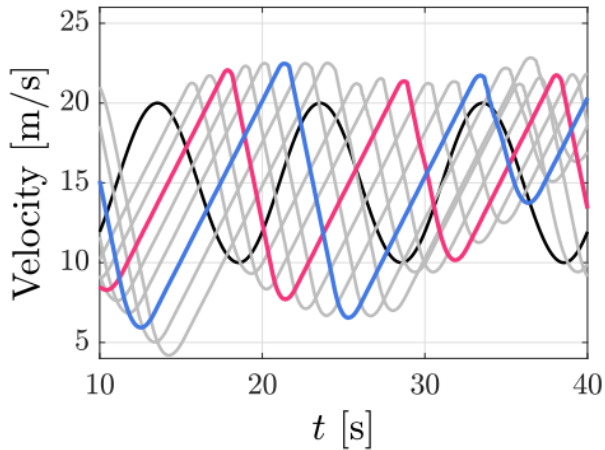
$$\min_{g, u, y, \sigma_y} \|y\|_Q^2 + \|u\|_R^2 + \lambda_g \|g\|_2^2 + \lambda_y \|\sigma_y\|_2^2$$

$$\text{s.t.} \quad \begin{bmatrix} U_p \\ E_p \\ Y_p \\ U_f \\ E_f \\ Y_f \end{bmatrix} g = \begin{bmatrix} u_{ini} \\ \epsilon_{ini} \\ y_{ini} \\ u \\ \epsilon \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \sigma_y \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

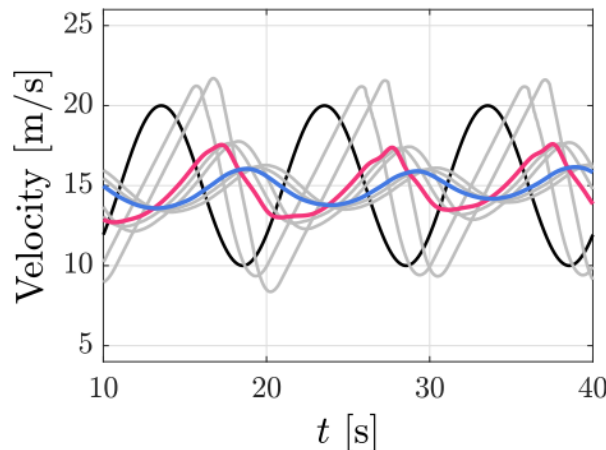
$$\epsilon = 0, u \in \mathcal{U}, y \in \mathcal{Y}.$$

# Experiments

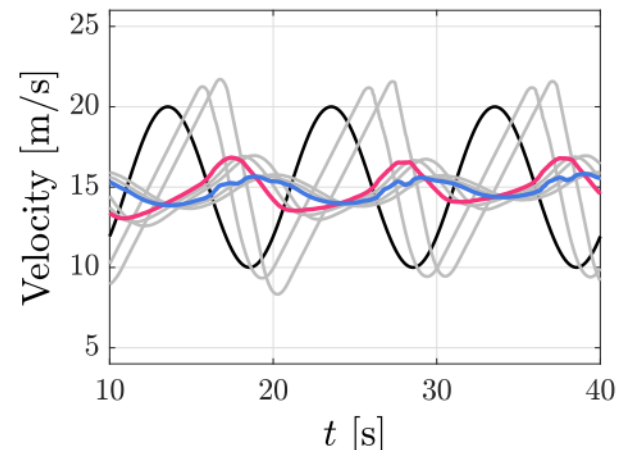
Comparison with MPC (Head Vehicle  $\rightarrow$  2 HDVs  $\rightarrow$  1 CAV  $\rightarrow$  2 HDVs  $\rightarrow$  1 CAV  $\rightarrow$  2 HDVs)



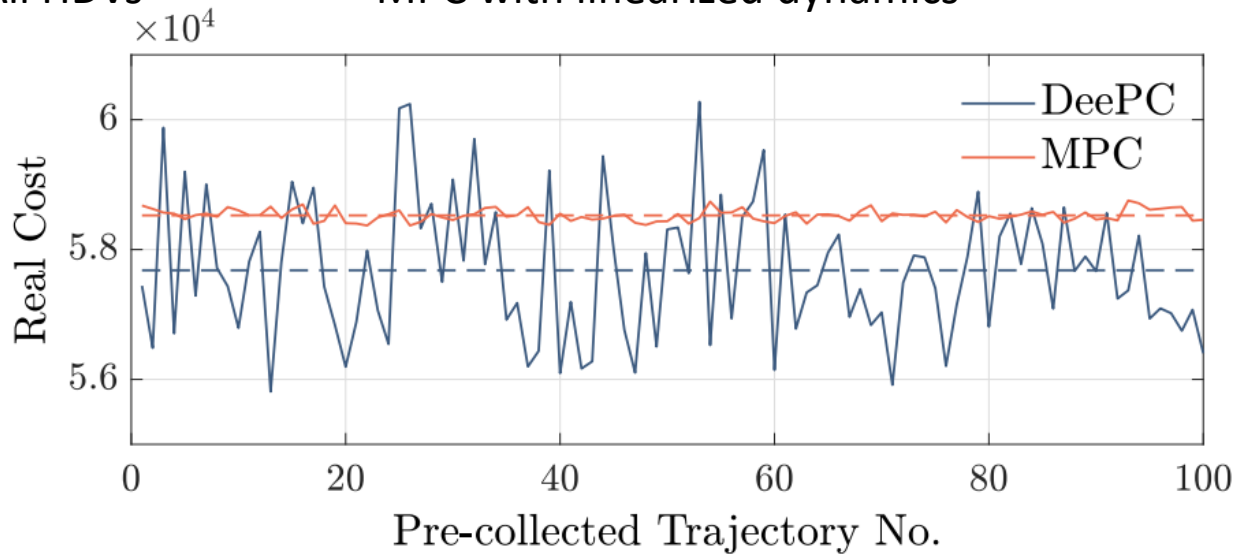
All HDVs



MPC with linearized dynamics



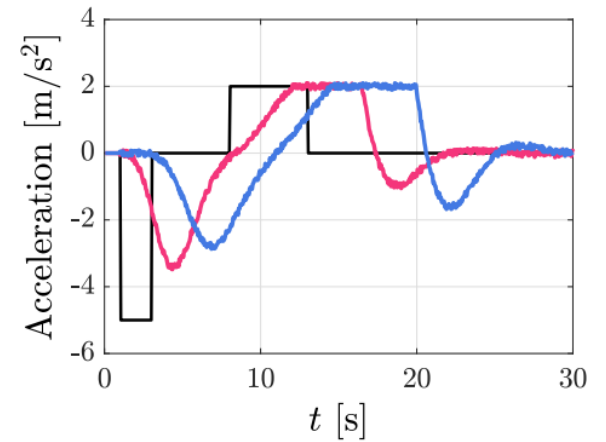
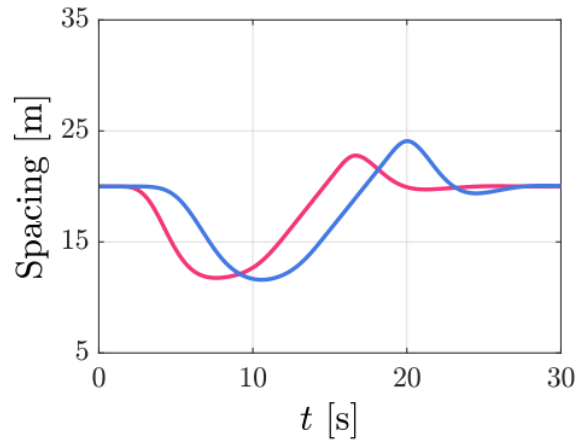
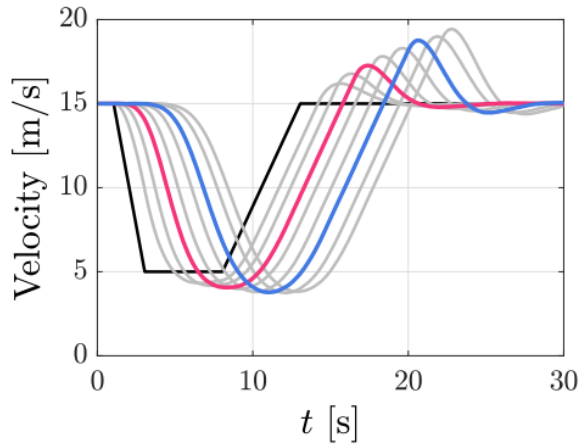
DeePC



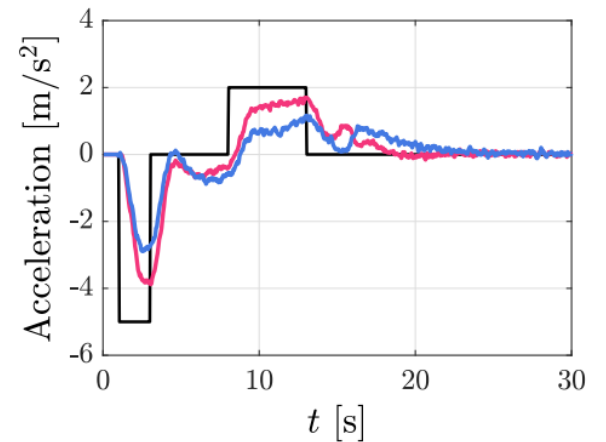
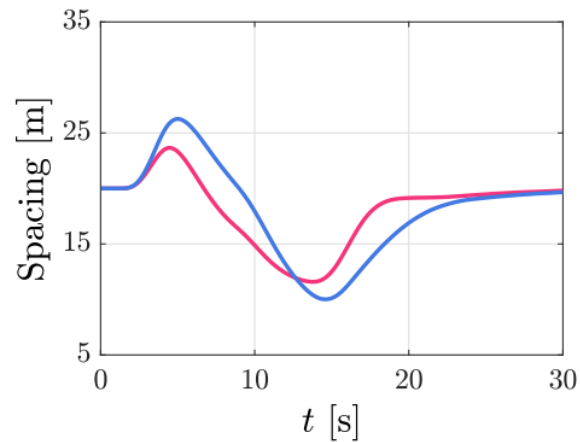
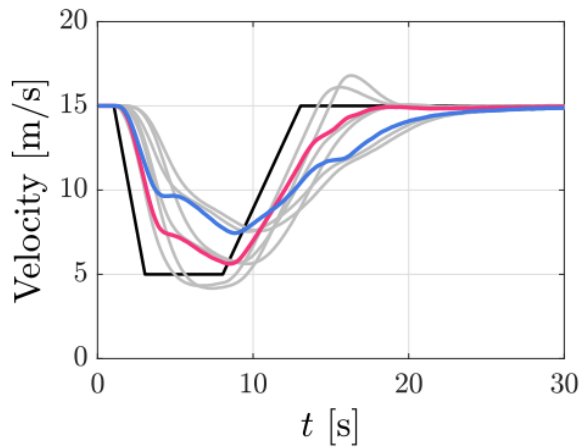


# Experiments

## Simulation at safety-critical scenario



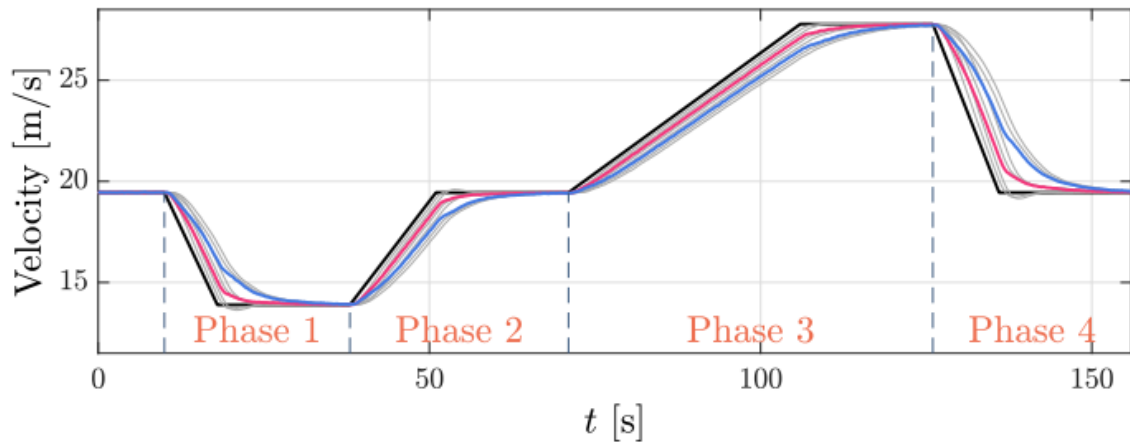
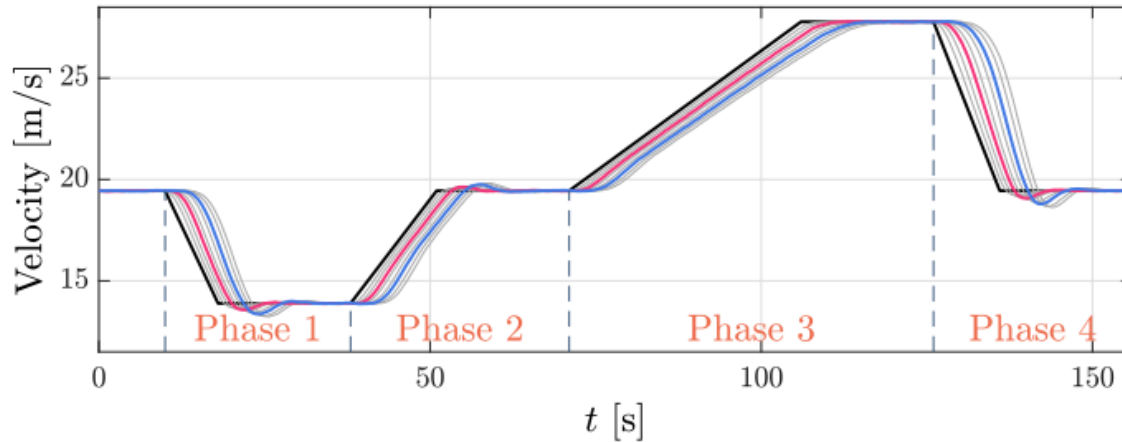
All HDVs



DeePC

# Experiments

## Comprehensive simulation



---

### Fuel consumption reduction

---

Phase 1	7.59%
Phase 2	1.20%
Phase 3	0.53%
Phase 4	5.39%

---

### Tracking error reduction

---

Phase 1	15.16%
Phase 2	12.08%
Phase 3	4.52%
Phase 4	11.32%

---

# Modeling and Control of Traffic Flow

- **Modeling techniques**

- Ordinary differential equations
- Partial differential equations
- Queuing theory
- Cell transmission model
- Cascaded nonlinear systems
- etc.

Horowitz, R., & Varaiya, P. Control design of an automated highway system. *Proc. IEEE*, 2000.

Bellomo, N., & Dogbe, C. On the modeling of traffic and crowds: A survey of models, speculations, and perspectives. *SIAM review*, 2011

Geroliminis, N., & Daganzo, C. F. Macroscopic modeling of traffic in cities. *In Transportation Research Board 86th Annual Meeting*, 2007.

Daganzo, C. F. The cell transmission model, part II: network traffic. *TRB*, 1995.

Helbing, D. Traffic and related self-driven many-particle systems. *Reviews of modern physics*, 2001.

- **Control methods**

- Adaptive control
- Model predictive control
- Optimal cooperative control
- Reinforcement Learning
- Formal methods
- etc.

Hegyi, A., De Schutter, B., & Hellendoorn, H. Model predictive control for optimal coordination of ramp metering and variable speed limits. *TRC*, 2005

Prashanth, L. A., & Bhatnagar, S. . Reinforcement learning with function approximation for traffic signal control. *IEEE TITS*, 2010.

Smulders, S. Control of freeway traffic flow by variable speed signs. *TRB*, 1990.

Haddad, J., Ramezani, & Geroliminis, N. Cooperative traffic control of a mixed network with two urban regions and a freeway. *TRB*, 2013